

WAVE DISTURBANCES FROM LOCALIZED SOURCES AT THE ICE–WATER ENVIRONMENT BOUNDARY

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Abstract: Floating ice cover determines dynamic interaction between the ocean and the atmosphere, influencing the dynamics of the sea surface and subsurface waters, as the ice cover and the entire mass of water beneath it participate in the overall vertical motion. This paper examines the amplitude-phase structure of wave fields arising at the interface between ice and homogeneous water of finite thickness, generated by moving and stationary localized disturbance sources. The ice cover is modeled as a thin elastic plate with small deformations, while the plate is physically linear. Integral representations of the solution are obtained in the linear approximation, and the results of dispersion relations and phase patterns are presented for various wave generation parameters. It is shown that the main parameters determining the characteristics of the amplitude-phase structure of wave disturbances on the ice surface are ice thickness, velocity of the wave source, and oscillation frequency. Numerical calculations demonstrate that changes in source velocity, ice thickness, and oscillation frequency lead to a noticeable qualitative restructuring of the phase patterns of the excited far-field wave fields at the ice–water interface. The wave pattern of disturbances far from the appearing source of disturbances is a wave system of long- and short-wave packets, with shorter waves traveling at higher velocities and longer wave trains propagating at slower velocities. The spatial and temporal evolution of wave packets of ice surface disturbances propagating from the appearing source of disturbances is investigated.

Keywords: ice cover, interface elevation, dispersion dependencies, amplitude-phase structure, localized source

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1. Introduction

A characteristic natural factor of the polar regions of the World Ocean and freezing sea regions is the presence of ice cover [Badulin et al., 2025; Bukatov, 2017; Bulatov and Vladimirov, 2024, 2025; Jiang et al., 2023; Ni et al., 2025; Squire et al., 1996; Sturova and Tkacheva, 2025]. Floating ice cover, which determines the dynamic interaction between the ocean and the atmosphere, influences the dynamics of not only the sea surface but also subsurface waters, with both the ice cover and the entire mass of water beneath it participating in the overall vertical motion. Wave motion at the interface of the ice cover in marine conditions can arise both due to natural causes or may be caused by a flow around artificial obstacles, such as submerged parts of hydrocarbon production platforms, offshore pipelines, and other complex hydraulic structures. In real ocean conditions, ice surface deformation can be caused by, for example, pulsing and periodic pressure changes, underwater sources of various physical origin (including underwater explosions), loads of constant and variable intensity moving across the ice, and localized variations in the sea surface. Wave action can explain phenomena such as crack formation in continuous ice fields, ice breakup in marginal zones, and fast ice breakup [Squire et al., 1996; Sturova and Tkacheva, 2025].

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One of the significant sources of ice cover disturbances can be intense packets of internal gravity waves: the amplitudes of ice cover disturbances due to such waves can range from centimeters to several meters [Bukatov, 2017; Marchenko *et al.*, 2013; Marchenko and Morozov, 2016; Marchenko *et al.*, 2012; Morozov *et al.*, 2019; Sahoo *et al.*, 2025]. Studying wave processes in the sea with floating ice cover is relevant for understanding its response to various hydrodynamic disturbances, moving surface and underwater vessels, ice floe breakup processes for shipping purposes, and improving methods for remote sensing of the ice cover surface. The practical interest in the impact of ice cover on underwater obstacles is also because in the presence of a water column an obstacle flown around by the flow generates waves at the interface between the ice and the marine environment and, consequently, experiences additional wave resistance, the calculation of which is necessary when designing various structures [Bukatov, 2017; Chowdhury and Mandal, 2006; Das *et al.*, 2018; Pogorelova *et al.*, 2019; Sturova and Tkacheva, 2025]. On the other hand, these surface disturbances carry information both about the sources of the disturbances themselves and about the characteristics of the marine environment beneath the ice; they can be recorded using specialized equipment, primarily radar and optical systems.

The development of methods for monitoring ice-covered marine waters based on the data of remote sensing of the ice surface presupposes, in particular, the possibility of effectively calculating the force impact of sea currents on underwater sources of various physical origin, taking into account flow velocity, ice thickness, source immersion depth, its variability parameters, and other significant hydrodynamic characteristics. The generation of wave disturbances at the ice–water interface from underwater obstacles in flowing water has been the subject of numerous studies, both in laboratory experiments and within the framework of theoretical papers. The current state of the problem and a review of the research can be found in [Badulin *et al.*, 2025; Das *et al.*, 2018; Dinvay *et al.*, 2019; Jiang *et al.*, 2023; Ni *et al.*, 2025; Sahoo *et al.*, 2025; Sturova and Tkacheva, 2025].

Using model representations to describe non-local sources, wave disturbances at the ice–water boundary in the far zone can be described by relatively simple analytical formulas. A typical approach to study the wave pattern generated by various sources in the linear approximation is to represent the wave field through the Fourier transform and then construct the asymptotics of the corresponding integrals in the far zone [Bulatov and Vladimirov, 2023; Bulatov *et al.*, 2023, 2025; Hosking and Milinazzo, 2022; Lu and Dai, 2008; Lu *et al.*, 2008; Malenko and Yaroshenko, 2023; Sturova, 2021]. The possible sources are: mechanical destruction of glaciers accompanied by icequakes, iceberg calving, sharp motions of ice as it slides into the water from the underlying sedimentary base near the grounding line of glaciers, fluctuations in the atmospheric pressure, motion of an external load, which can be a regular vehicle, an aircraft during takeoff or landing, hovercraft, motion of a submerged body, water flow around bottom irregularities, bottom displacement, underwater earthquakes, surface or underwater explosions [Marchenko *et al.*, 2013; Marchenko and Morozov, 2016; Marchenko *et al.*, 2012; Morozov *et al.*, 2019; Pogorelova *et al.*, 2019; Sahoo *et al.*, 2025; Squire *et al.*, 1996; Stepanyants and Sturova, 2021].

Numerous laboratory experiments and field observations in the World Ocean demonstrate that the ice cover in marine conditions behaves like a thin elastic plate floating on the surface of water [Badulin *et al.*, 2025; Hossain and De, 2024; Ilyichev, 2021; Ilyichev *et al.*, 2023; Sahoo *et al.*, 2025]. Therefore, this paper utilizes a model of an elastic plate, for which, without loss of physical generality, it can be assumed that its small deformations satisfy Hooke's law. This means that each elementary volume of the elastic ice layer, which is orthogonal to the neutral surface in the undeformed state, remains straight and orthogonal to this surface after deformation. It is also assumed that the ice cover is continuous, that is, its horizontal scales exceed the lengths of the excited waves and, under sufficient natural conditions, is modeled by a thin elastic plate whose deformations are small and the plate is physically linear [Dinvay *et al.*, 2019; Hosking and Milinazzo, 2022; Ilyichev, 2021; Ilyichev *et al.*, 2023; Sahoo *et al.*, 2025; Squire *et al.*, 1996].

The aim of this paper is an analytical and numerical study of the amplitude-phase structures of wave fields arising at the interface between ice and a homogeneous liquid of finite thickness, generated by moving and stationary localized disturbance sources.

2. Oscillating Moving Source: Problem Formulation, Integral Forms of Solutions, Dispersion Dependencies, and Phase Structure of Wave Fields

We consider a flow of ideal fluid of depth H around a point pulsating source of mass with intensity $Q \exp(i\omega t)$ ($Q = \text{const}$), where ω is the oscillation frequency. The fluid is bounded from above by an ice cover of thickness h . The horizontal plane xy coincides with the unperturbed interface between the fluid of density ρ_0 and ice of density ρ_1 . The velocity of the source in a fluid of finite thickness is directed along the x -axis and is equal to V ; the source is located at depth z_0 , $-H < z_0 < 0$. We denote the velocity perturbation potential as $\Phi(x, y, z, t, z_0)$: $\nabla\Phi = (u, v, w)$, and the elevation of the interface between the fluid and the ice cover as $\Lambda(x, y, t, z_0)$. Then, $(V + u, v, w)$ is the velocity vector of an arbitrary fluid particle, and the solution is considered in a coordinate system moving together with the source. Functions $\Phi(x, y, z, t, z_0)$, $\Lambda(x, y, t, z_0)$ are sought in the form: $\Phi(x, y, z, t, z_0) = \exp(i\omega t)\varphi(x, y, z, z_0)$, $\Lambda(x, y, t, z_0) = \exp(i\omega t)\eta(x, y, z_0)$. Then, in the linear approximation, the mathematical formulation of the problem is formulated as follows [Bulatov and Vladimirov, 2023, 2024, 2025; Bulatov et al., 2025; Savin and Savin, 2015]:

$$\left(\Delta + \frac{\partial^2}{\partial z^2}\right)\varphi = Q\delta(x)\delta(y)\delta(z - z_0) \quad (1)$$

$$\frac{D\varphi}{Dt} + g\eta - C\Delta\eta + B\Delta^2\eta + A\frac{D^2\eta}{Dt^2} = 0, \quad \frac{D\eta}{Dt} = \frac{\partial\varphi}{\partial z}, \quad \text{at } z = 0; \quad \frac{\partial\varphi}{\partial z} = 0, \quad \text{at } z = -H$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\frac{D}{Dt} = i\omega + V\frac{\partial}{\partial x}$, $A = \frac{h\rho_1}{\rho_0}$, $B = \frac{Eh^3}{12\rho_0(1-\nu_0^2)}$, $C = \frac{\sigma h}{\rho_0}$, g is the acceleration due to gravity, E is Young's modulus of ice, ν_0 is Poisson's ratio, σ is the initial stress. The characteristic values of these parameters in sea conditions are [Sahoo et al., 2025; Squire et al., 1996]: $\rho_0 = 1025 \text{ kg/m}^3$, $\rho_1 = 0.9\rho_0$, $E = 3 \times 10^9 \text{ N/m}^2$, $\nu_0 = 0.3$, $\sigma = 10^5 \text{ N/m}^2$. Solution of problem (1) is sought using the Fourier method. Then, the integral representation of the ice cover elevation $\Lambda(x, y, t, z_0)$ is written as [Bulatov and Vladimirov, 2023, 2025; Bulatov et al., 2023, 2025]:

$$\Lambda(x, y, t, z_0) = -2\pi i \sum_{j=1}^J \int_{L_j(\alpha)} \frac{F(\omega, \mu_j(\nu), \nu, z_0)}{T(\omega, \mu_j(\nu), \nu)} \exp(-i\nu y - i\mu_j(\nu)x) d\nu \quad (2)$$

$$T(\omega, \mu_j(\nu), \nu) = \frac{\partial Z(\omega, \mu_j(\nu), \nu)}{\partial \mu}, \quad Z(\omega, \mu_j(\nu), \nu) = (\mu_j(\nu)V - \omega)^2 - \Omega^2(k)$$

$$F(\omega, \mu_j(\nu), \nu, z_0) = \frac{iQ \exp(i\omega t)}{4\pi^2} \frac{(\mu_j(\nu)V - \omega) \cosh(k(z_0 + H))}{\cosh(kH)(A k \tanh(kH) + 1)}$$

$$\Omega^2(k) = \frac{k(g + Ck^2 + Bk^4) \tanh(kH)}{A k \tanh(kH) + 1}, \quad k^2 = (\mu_j(\nu))^2 + \nu^2$$

where $\Omega^2(k)$ is the dispersion relation for waves at the interface between the layer of homogeneous fluid with finite depth H at rest and ice [Bulatov and Vladimirov, 2023, 2025; Savin and Savin, 2015; Sturova and Tkacheva, 2025]. Function $Z = Z(\omega, \mu, \nu)$ is formulated so that equation $Z = 0$ is one of the possible forms of dispersion relation. $\mu_j(\nu)$ ($j = 1, 2, \dots, J$) are roots of equation $Z(\omega, \mu_j(\nu), \nu) = 0$ (summation is performed over all roots of this equation), $L_j(\alpha)$ are those intervals of the dispersion curves where inequality $\frac{\partial\omega}{\partial\mu} > 0$ is true at $\mu = \mu_j(\nu)$.

The maps of dispersion curves $\mu_j(\nu)$, which are the corresponding solutions of equation $Z(\omega, \mu, \nu) = 0$, have a rather complex topology and depend significantly on such problem

parameters as the velocity of the disturbance source, frequency of its oscillations, and thickness of the ice cover [Bulatov and Vladimirov, 2023; Bulatov et al., 2023, 2025; Sahoo et al., 2025]. Numerical results are presented for parameters characteristic of a real marine environment: ice thickness $h = 0.25$ m, water depth $H = 100$ m. Three critical values $V_{0,1,2}$ ($V_0 < V_1 < V_2 = \sqrt{gH}$) of the source velocity exist in this problem, which determine qualitatively different behavior of the dispersion relations and, hence, the spatial properties of the phase patterns of the generated wave fields. The value of V_0 is determined by the minimum phase velocity of the waves at the interface of a stationary fluid of finite thickness under ice: $V_0 = \frac{\Omega(k_0)}{k_0}$, where k_0 is the only positive root of equation: $2ABk^5 + 3Bk^4 + Ck^2 - 2Agk - g = 0$ [Bulatov and Vladimirov, 2023, 2025; Sahoo et al., 2025; Savin and Savin, 2015; Sturova and Tkacheva, 2025]. Value V_1 corresponds to the velocity of the source V , at which dispersion curves specified parametrically: $\mu_j(\nu) = \frac{\Omega(k)}{V}$, $\nu_j(k) = \sqrt{k^2 - (\frac{\Omega(k)}{V})^2}$, have no inflection points at $V < V_1$, and have at least two pairs of inflection points at $V > V_1$. This means that at $V < V_1$, equation $\mu'_j(k)\nu''_j(k) - \nu'_j(k)\mu''_j(k) = 0$ has no real roots, and at $V > V_1$ it has at least two real roots. The values of critical velocities for the parameters of this problem are equal to: $V_0 = 8.91$ m/s, $V_1 = 14.56$ m/s, $V_2 = 31.31$ m/s.

If $V < V_0$, wave oscillations do not exist at $\omega = 0$. If $V_0 < V < V_1$, the dispersion curve at $\omega = 0$ is convex and does not cover the origin of coordinates $\mu = \nu = 0$. If $V_1 < V < V_2$, the dispersion curve at $\omega = 0$ is not convex, it has two inflection points and does not cover the origin of coordinates $\mu = \nu = 0$. At $\omega \neq 0$, dispersion curves may consist of two or three closed curves; each of the dispersion relations $\mu_j(\nu)$ ($j = 1, 2, 3$) is one closed branch. At $\omega < \omega^- = \Omega(k^-) - k^-V = 0.241$ s $^{-1}$ ($k^- = 0.0266$ m $^{-1}$ is the smallest root of equation $\frac{\partial \Omega}{\partial k} = V$), both the lower $\mu_1(\nu)$ ($\mu_1(\nu) < 0, \forall \nu$) and middle $\mu_2(\nu)$ branches of the dispersion curves exist. When ω increases from zero to ω^- , these two branches expand towards each other and at $\omega = \omega^-$ the two branches merge into one lower branch. During further increases in frequency ω expansion of the remaining one branch occurs. The upper branches of dispersion curves $\mu_3(\nu)$ exist only at those frequencies ω , which satisfy inequality $\omega < \omega^+ = k^+V - \Omega(k^+) = 0.192$ s $^{-1}$ ($k^+ = 0.181$ m $^{-1}$ is the largest root of equation $\frac{\partial \Omega}{\partial k} = V$). When ω increases from zero to ω^+ narrowing of this branch of the dispersion curve occurs and at $\omega = \omega^+$ it degenerates into a point. Quantitative behavior of the dispersion curves at $V_1 < V < V_2$ remains the same, but at $\omega = 0$, the lower and upper branches of the dispersion curves are non-convex. At $V > V_2$, only two branches of the dispersion curves exist: the lower (encompassing point $\mu = \nu = 0$) and the upper.

Figures 1–6 show the results of calculations of phase patterns, which reveal that, depending on the main parameters of wave generation, one to three wave systems are excited simultaneously, related to the corresponding branches of the dispersion relations. Figures show the results of calculations of equal phase lines for $V = 10$ m/s (Figure 1: $\omega = 0.25$ s $^{-1}$, Figure 2: $\omega = 0.3$ s $^{-1}$, Figure 3: $\omega = 0.5$ s $^{-1}$, Figure 4: $\omega = 1$ s $^{-1}$); in Figures 5 and 6, $V = 35$ m/s (Figure 5: $\omega = 0$, Figure 6: $\omega = 2$ s $^{-1}$).

Figures 1–3 and 5 show a relatively complex wave pattern consisting of longitudinal (wedge-shaped) and transverse (ring) waves propagating from the source of disturbances both upstream and downstream; the direction of propagation of each wave system depends on the sign of the dispersion curves for any values of ν . Solid lines in the figures are lines of equal phase with a step of 2π . Dashed lines in the figures are wave fronts (caustics), the equations of which are written as $x = \pm \mu'(\nu_{1,2}^*)y$, where $\nu_{1,2}^*$ are two roots of equation $\mu''(\nu_{1,2}^*) = 0$. Wave fronts define the spatial region where wave oscillations of the ice–water interface exist. The green and red solid lines in the figures are defined as $x = \pm \mu'(\nu_{1,2}^0)y$, where $\nu_{1,2}^0$ are two roots of equation $\mu'(\nu_{1,2}^0) = \frac{\mu(\nu_{1,2}^0)}{\nu_{1,2}^0}$, the values of phase on these lines at stationary points in the integral representations (2) are equal to zero. Long longitudinal waves originate from the caustics; they are parallel to the green rays, and propagate upstream. Short longitudinal waves are parallel to the red rays and also propagate upstream.

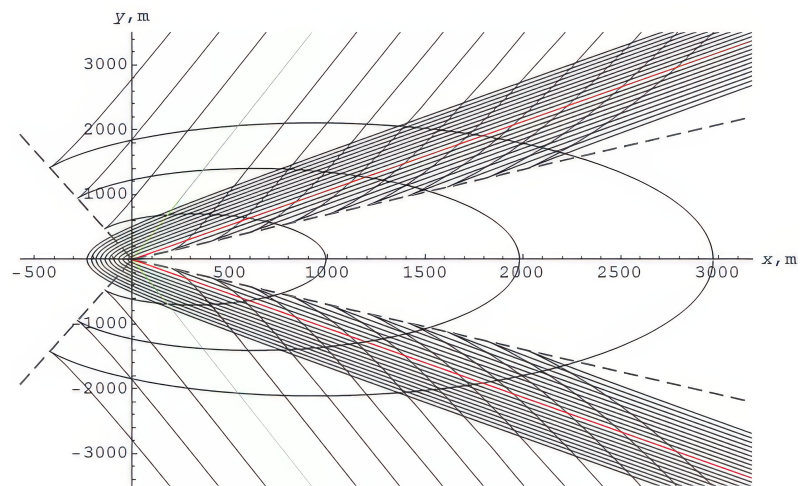


Figure 1. The wave system of two types: transverse (ring) waves propagate downstream, longitudinal (wedge-shaped) waves propagate upstream; two wave fronts are located behind the moving source at $x > 0$, two wave fronts are located ahead of the moving source at $x < 0$.

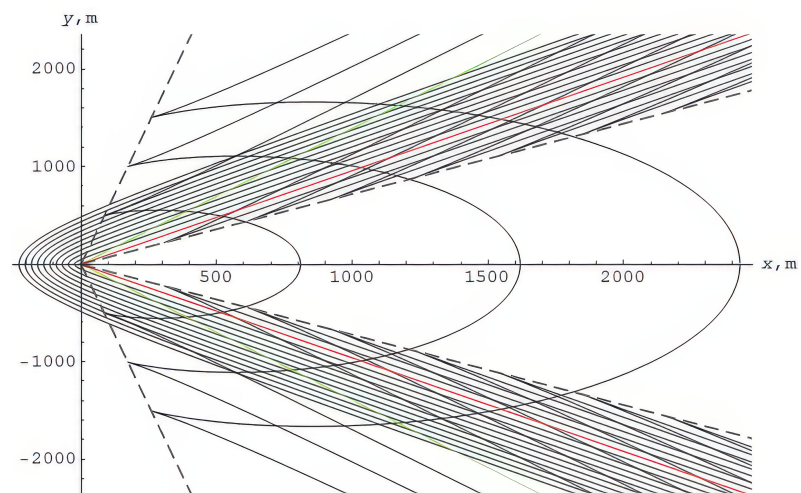


Figure 2. The wave system of two types: transverse (ring) waves propagate downstream, longitudinal (wedge-shaped) waves propagate upstream; four wave fronts are located behind the moving source at $x > 0$.

In Figures 4 and 6, the wave pattern represents a system of only transverse (ring) waves propagating downstream from the source. Numerical calculations also demonstrate a noticeable rearrangement of the phase pattern topologies with changes in the oscillation frequency ω of the moving source. It can be noted, in particular, that at certain values of frequency ω , wave fronts (precursors) exist that propagate ahead of the moving source of disturbances.

Numerical calculations also show that when the key wave generation parameters (source velocity, ice thickness, and oscillation frequency of the source) change, the wave pattern can consist of several wave types, and a noticeable qualitative restructuring of the phase patterns of the excited wave fields at the ice–water interface occurs. Therefore, the increased complexity of the observed wave patterns of ice cover elevation, including the appearance of additional wave structures, may be one indication of a significant change in such marine environment properties as source velocity, ice thickness, and oscillation frequency. Dispersion relations $\mu(\nu)$ can be closed, everywhere convex curves, and can also have two pairs of inflection points, which exist only at sufficiently small values of wave numbers and are located symmetrically with respect to the $\nu = 0$ axis. This complication of the topology of dispersion dependencies leads to the generation of an additional system of

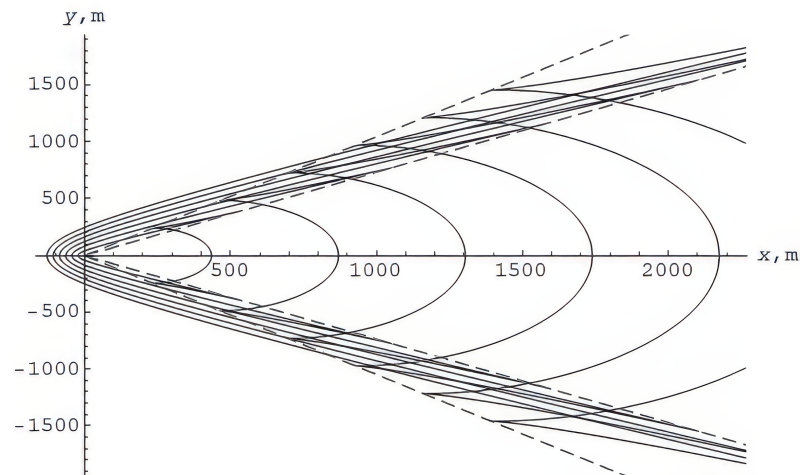


Figure 3. The wave system of two types: transverse (ring) waves propagate downstream, longitudinal (wedge-shaped) waves propagate upstream; four wave fronts are located behind the moving source at $x > 0$.

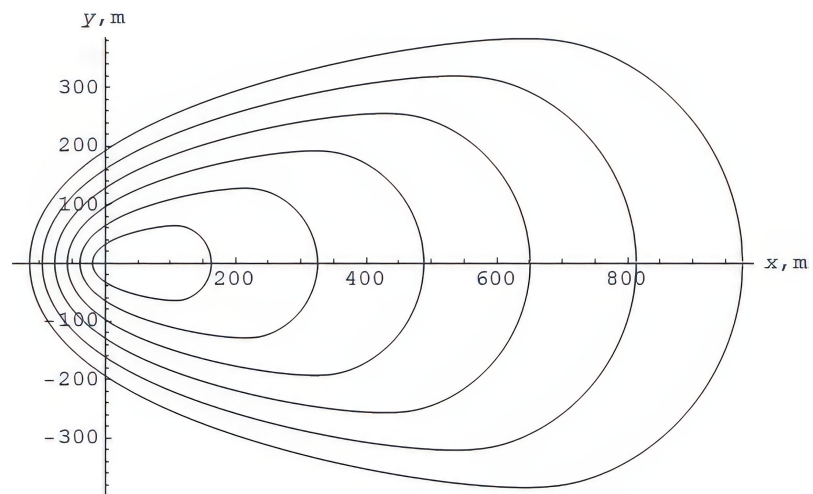


Figure 4. The wave system of transverse (ring) waves propagates in all directions from a moving source.

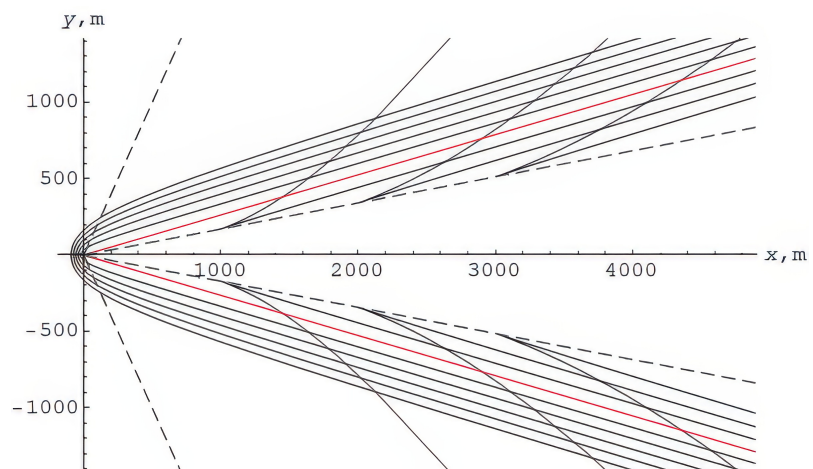


Figure 5. A wave system of longitudinal (wedge-shaped) waves propagates downstream; two wave fronts are located behind the moving source at $x > 0$.

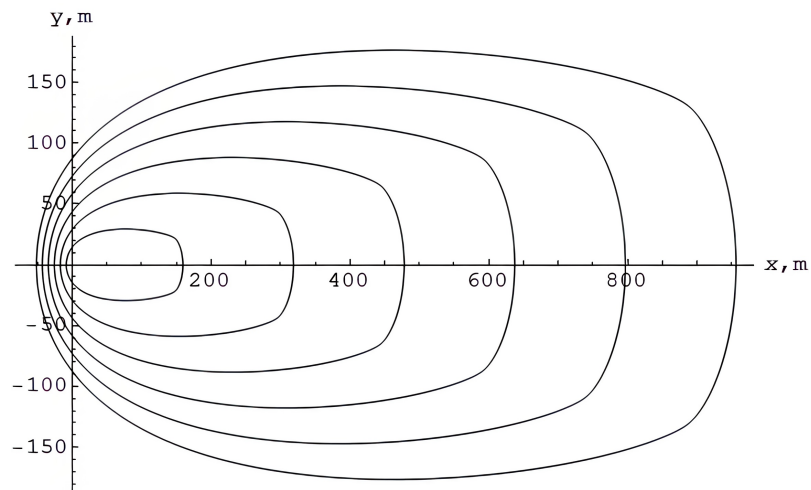


Figure 6. The wave system of transverse (ring) waves propagates in all directions from a moving source.

transverse waves and the appearance of corresponding pairs of wave fronts (dashed lines in the figures); in this case, phase patterns can demonstrate spatial structures of the “dovetail” type, when a qualitative restructuring of simultaneously arriving wave fronts occurs at a fixed observation point. Local extrema of the dispersion relations $\mu'(v)$ are most interesting from a practical standpoint since the asymptotic behavior of the far-field wave fields in the vicinity of the corresponding wavefronts and caustics corresponding to these extrema can be described using the method of standard integrals. The complexity of the topology of the calculated dispersion relations requires the use of specialized mathematical apparatus for a correct asymptotic study of the excited wave fields.

An increase in current velocity at constant ice thickness and oscillation frequency leads to a broadening (in wavenumber space) of the dispersion curves. The curve corresponding to a lower flow velocity is entirely within the curve corresponding to a higher flow velocity. Therefore, as the source velocity V increases, the wavelength along the positive direction of the x -axis increases, and decreases along the negative direction of the x -axis.

As the source velocity increases, the spatial region where wave oscillations exist decreases. Outside this region, the amplitudes of far-field wave fields are exponentially small. The same effect is observed with changing ice thickness at constant velocity of the source. As ice thickness increases, while the source velocity and oscillation frequency remain constant, the dispersion curves narrow (in wavenumber space), and, correspondingly, the spatial region of wave oscillations expands. The wavelength increases along the positive x -axis, while it decreases along the negative x -axis.

3. Appearance of a Stationary Source: Problem Formulation, Integral Forms of Solutions, and Amplitude-Phase Structure of Wave Fields

In this section, we study wave disturbances on the interface between an ice cover and a layer of homogeneous water, arising from a source of mass located at point $(0, 0, z_0)$, $-H < z_0 < 0$, which appeared at time $t = 0$ and instantly ejected a volume of water Q . Here, Φ is the velocity disturbance potential: $\nabla\Phi = (u, v, w)$, which is determined from the following problem [Bulatov and Vladimirov, 2023, 2025; Bulatov et al., 2023, 2025]:

$$\left(\Delta + \frac{\partial^2}{\partial z^2}\right)\Phi(x, y, t, z, z_0) = Q\delta(t)\delta(x)\delta(y)\delta(z - z_0) \quad (3)$$

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} - C\Delta\left(\frac{\partial\Phi}{\partial z}\right) + B\Delta^2\left(\frac{\partial\Phi}{\partial z}\right) + A\frac{\partial^2}{\partial t^2}\left(\frac{\partial\Phi}{\partial z}\right) = 0, \quad \text{at } z = 0; \quad \frac{\partial\Phi}{\partial z} = 0, \quad \text{at } z = -H$$

$$\Phi \equiv 0, \quad \text{at } t < 0$$

Problem (3) has been solved using the Fourier method describing elevation of the ice cover $\eta(x, y, t, z_0)$ ($\frac{\partial \eta}{\partial t} = \frac{\partial \Phi}{\partial z}$). The solution is written as [Bulatov et al., 2025]:

$$\eta(r, t, z_0) = \frac{Q}{2\pi} \int_0^\infty \frac{k \cosh(k(z_0 + H))}{(A k \tanh(kH) + 1) \cosh(kH)} J_0(kr) \cos(\Omega(k)t) dk \quad (4)$$

where J_0 is the Bessel function of order zero.

Figure 7 presents the results of calculations of the time evolution of a radially symmetric field of the ice–water interface elevation from a localized mass source that appeared at time $t = 0$. The calculation parameters, characteristic of real marine environment were as follows: $Q = 1000 \text{ m}^3$, $h = 0.25 \text{ m}$, $H = 50 \text{ m}$, $z_0 = -10 \text{ m}$.

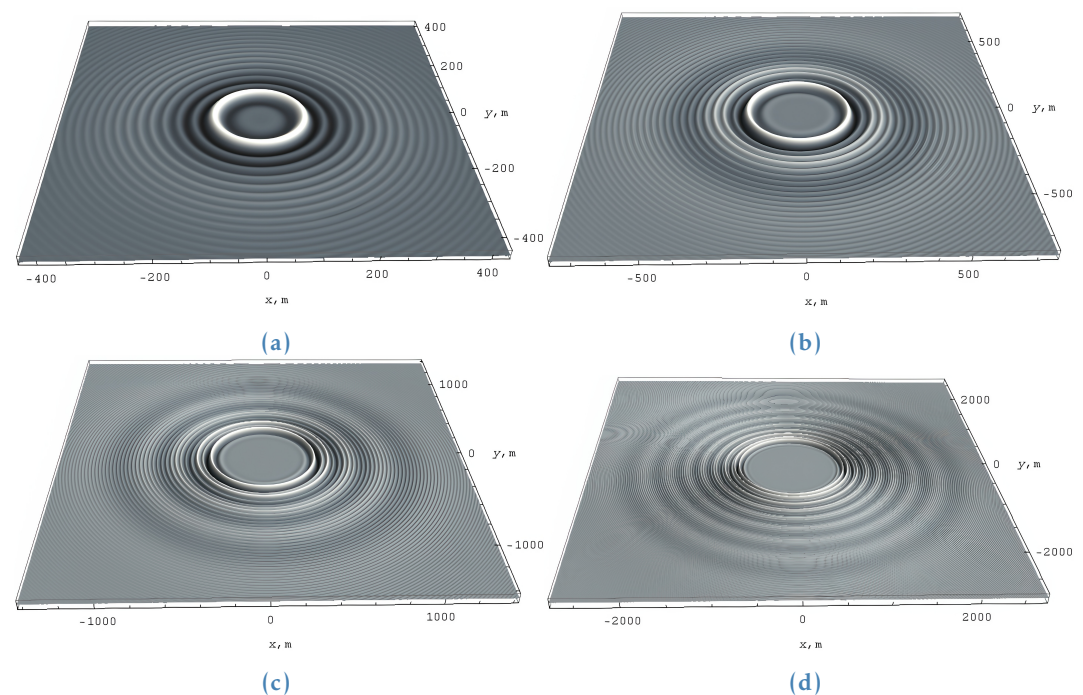


Figure 7. Temporal variability of ice–water interface elevation from an impulsive mass source: (a) $t = 15 \text{ s}$, (b) $t = 30 \text{ s}$, (c) $t = 60 \text{ s}$, (d) $t = 120 \text{ s}$.

The qualitative picture of wave disturbances far from the appearing source of disturbances is a wave system of long- and short-wavelength packets, with shorter waves traveling at higher speeds and longer wave trains propagating at a slower speed. Over sufficiently long times and distances from the appearing source of disturbances, a wave pattern of relatively high-frequency oscillations is clearly defined, modulated by comparatively low-frequency oscillations. In the problem considered here, long waves are those with a leading edge, while short waves lack it; and both types of waves share a common trailing edge. However, it can be said that short waves arrive at a fixed observation point before long waves, and the group velocity of short waves is greater than the velocity of the long-wave front; hence, short waves arrive at a fixed observation point before long waves.

Let us describe a qualitative picture of the time evolution of far fields of the elevation of the ice–water interface at a fixed observation point r , which are excited by a localized source appearing at time $t = 0$. This field is determined by the sum of two wave modes, each of which depends on the behavior of two stationary points of the phase function in the integral representation (4). The short-wave mode, corresponding to one stationary point, arrives at the observation point instantaneously, since the group velocity of short waves is unlimited from above. Its contribution to the total elevation field is of the order of $O(t^{-1})$. The wave field in the vicinity of the leading wave front is expressed through the product of the Airy function and its derivative, with the contribution of the long-wave mode corresponding to the second stationary point being exponentially small. At $t = r/\Omega'(0)$ the

long wave front passes through the observation point, beyond which the contributions of both modes to the total field are of the same order $O(t^{-1})$. Far from the wave fronts, the asymptotic behavior of the wave fields can be calculated using the stationary phase method. Let K be the value of the wave number k , at which the minimum group velocity of waves under the ice is gained, then the time moment $t = r/\Omega'(K)$ ($\Omega'(0) > \Omega'(K)$) for the given fixed value of r corresponds to the passage of the trailing wave front. The wave field in the vicinity of the trailing wave front is expressed through the Airy function and decays as $O(t^{-5/6})$. Behind this front, the elevation field is exponentially small, since each wave mode making up the total field is individually exponentially small.

At each fixed and sufficiently large moment of time t , the radially symmetric wave pattern has the following qualitative character as the distance from the disturbance source increased. At $r < t\Omega'(K)$, the entire wave field is exponentially small. The value $r = t\Omega'(K)$ corresponds to the trailing wave front, beyond which the wave field is exponentially small. The value $r = t\Omega'(0)$ corresponds to the leading edge of long waves. Between the leading and trailing edges, the contributions of each wave mode to the total field are of the order of $O(r^{-1})$. At $r > t\Omega'(0)$, the short-wave mode contributes of the order of $O(r^{-1})$, and the contribution of the long-wave mode is exponentially small.

4. Conclusions

In this paper, using a linear approximation, an integral representation of the solution was obtained for describing the wave fields of small surface disturbances of the ice cover from a stationary and moving localized disturbance source. The topology of the main dispersion relationships was studied, allowing for the analysis of the amplitude-phase structure of the wave fields both near and far from the wave fronts. The spatial and temporal evolution of ice cover disturbance wave packets far from the sources was analyzed analytically and numerically. In the future, for solving a number of important practical problems and applications, using the convolution operation, it will be possible to calculate ice cover wave disturbances generated by spatially distributed nonlocal and nonstationary disturbance sources. The obtained results enable the study of ice cover surface disturbances, which can be recorded using radar and optical systems. They provide information not only about the sources of generation but also about the characteristics of the marine environment beneath the ice. This is important, among other things, for studying the ice cover's response to various hydrodynamic disturbances, investigating ice floe decay processes, and improving methods for remote sensing of ice cover surfaces. The obtained results significantly simplify analytical representations of solutions, address qualitatively more complex problems, and, for example, adjust individual initial condition parameters based on in situ observations of ice floes under natural conditions.

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