

HIGH RESOLUTION SEISMICITY SMOOTHING METHOD FOR SEISMIC HAZARD ASSESSMENT

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Abstract: A high resolution smoothing method is proposed for performing local estimates of the parameters of the Gutenberg-Richter law (GR). Using this method, the smoothing radius can be chosen large enough to ensure that the condition of applicability of GR law is met, while the distinguished areas of high activity align well with the distribution of epicenters and there is no "smearing" of narrow areas of really high seismic activity into wider zones, which are not actually active at the edges.

Keywords: seismicity, seismic hazard, smoothing method, Gutenberg-Richter law, interpolation.

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1. Introduction

The problem of reliable assessment of seismic hazard and its application in practice has intensified after the catastrophic earthquakes on February 6 and 7, 2023 $M_w = 7.8$ and 7.5 on the border of Turkey and Syria and the earthquake on September 8, 2023 $M_w = 6.8$ in Morocco. In Turkey, like in Russia and in most countries located in earthquake-prone regions, seismic hazard assessments are used in earthquake-resistant construction standards (building codes) in order to minimize the number of casualties and economic losses in case of a devastating earthquake. Unfortunately, as it has happened in most cases of catastrophic earthquakes [Wyss and Kossobokov, 2012], the seismic hazard in the epicentral zones of these earthquakes is significantly underestimated.

Seismic hazard assessment is understood as an assessment of the probability of exceeding a seismic effect of a certain value in a certain location over a certain time period. The impact effect is usually measured in terms of macroseismic intensity or in units of ground acceleration. There are two main approaches to assessing seismic hazard – deterministic and probabilistic. In the deterministic approach, the possibility of exceeding a certain effect over a very long time interval (tens of thousands of years) is assessed. This approach is usually based on active tectonic faults data, paleo- and archaeoseismological studies, and data on instrumentally recorded large earthquakes. The goal of the probabilistic approach is to estimate the probability or recurrence of exceeding a certain effect over a certain time interval. Both, data on active faults and statistical analysis of instrumentally recorded earthquakes are typically used in this approach. Commonly, the abbreviations DSHA (Deterministic Seismic Hazard Analysis) and PSHA (Probabilistic Seismic Hazard Analysis) are used.

Usually, seismic hazard assessment is presented in the form of maps called seismic zoning maps. In Russia, depending on the detail of the scale, maps are classified as general (GSZ), detailed (DSZ) and microseismic (MSZ) zoning maps. GSZ maps are used on a state scale. The actual earthquake impact may depend on ground conditions (on rocky grounds,

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other things being equal, the impact is weaker than on soft or water-saturated soils), so GSZ maps are designed for average grounds.

The first seismic zoning maps were based on the deterministic approach, so traditionally GSZ maps are constructed in the form of zones of the maximum expected impact – macroseismic intensity in the MSK-64 scale [Medvedev *et al.*, 1964]. In the framework of PSHA, several versions of maps are constructed with estimates of the expected maximum impact for different time intervals (for example, 500, 5000, 10000 years).

Seismic hazard assessment consists of modeling the seismic regime or zones of possible earthquake sources (PES) and the impact of earthquakes on objects at a distance from the earthquake source. The seismic regime is a set of earthquakes considered in space and time, taking into account their magnitude [Riznichenko, 1958, 1968]; in fact, this is more general concept than PES zones. In PSHA, PES zones imply sources of earthquakes in the form of points (epicenters), lines (active faults), and areas (“domains”), which are considered as near-homogeneous seismogenic zones.

Nowadays, despite the apparent complexity, significant progress has been made in the world seismology in solving the problem of accurate modeling the impact of earthquakes compared to modeling PES zones. The errors in seismic hazard assessments are usually associated with inaccurate forecasting of the future earthquake source locations and their magnitudes. Many cases of earthquakes are known, that occurred in places where no one expected them to be such a magnitude. The most striking recent example is the magnitude $Mw9.0$ Tohoku earthquake near the coast of Japan on March 11, 2011. A very recent example is the September 8, 2023, $Mw = 6.8$, earthquake in Morocco.

In the USSR and then in the CIS many earthquakes were a surprise from the point of view of the GSZ maps used at that time: Gazli, 1976, $M = 7.0$ in Uzbekistan, Spitak, 1988, $M = 6.8$ in Armenia, Zaysan, 1990, $M = 6.9$ in Kazakhstan, Rachi, 1991, $M = 7.0$ in Georgia, Sususamy, 1992, $M = 7.5$ in Kyrgyzstan, Khailin, 1991, $M = 7.0$ in Koryak and Neftegorsk, 1995, $M = 7.6$ in Sakhalin. Large earthquakes occurred after the development of the GSZ-97 map set for the territory of Russia also demonstrated significantly larger impact than expected: Olyutor, 2006, $M = 7.6$ in Koryak, Tuva, 2012, $M = 6.9$, Ilin-Tass (Abyisky), 2013, $M = 6.7$ in Yakutia, Katav-Ivanovsk, 2018, $M = 5.4$ in the Urals, etc.

Another type of errors is a significant overestimation of seismic hazard over large areas. [Shebalin *et al.*, 2022] showed that in most regions of Russia the seismic hazard is overestimated by at least 10 times. The analysis was carried out by comparison of the areas expected in accordance with the map of the OSR-97A zones, and the total isoseismal areas from earthquakes that occurred in the period after the publication of the OSR-97 maps.

The main contribution to such errors is the overestimation of the recurrence rate of earthquakes over large areas. Thus, the inaccurate model of PES zones is the main source of errors in GSZ maps, both target misses and false alarms.

In modern world practice, as a rule, a combined seismic regime model is used for the purposes of general seismic zoning. The model includes seismicity along linear structures – “lineaments” (active tectonic faults), on area structures – “domains,” as well as individual potential earthquake sources. It is assumed that within a structure unit, seismic events have an equally probable epicenter location, and the same recurrence rate and frequency-magnitude distribution.

The transition from PES zones to GSZ maps is the construction of model of the earthquake impact propagated from the source to a point on the surface of the Earth may be performed either using a “classical” analytical method proposed by [Cornell, 1968] or by forming of a synthetic earthquake catalog. In the latter method at each location the impact of different magnitudes is calculated integrating the impacts of all synthetic events.

This approach was used to develop maps of the general seismic zoning of Russia GSZ-97, GSZ-2015, GSZ-2016, which were used as normative ones for design of earthquake-resistant construction in seismic regions. The model was created by a large group of authors under the leadership of V. I. Ulomov and was called the lineament-domain-focal (LDF) model [Ulomov and The GSHAP Region Working Group, 1999]. It should be noted that it is

incorrect to interpret the creation of such a model as an innovation first implemented in Russia, as it is often noted in Russian sources. At the international scope, this approach was used within the framework of the international project GSHAP (Global Seismic Hazard Assessment Program), operated from 1992 to 1999 [Giardini *et al.*, 1999]. Nevertheless, this model was a step forward in the practice of seismic hazard zoning. For the first time, probabilistic maps began to be used, intended for engineering of the earthquake-resistant constructions of different categories of vulnerability and length life.

However, the approach to design GSZ maps based on the LDF model can only conditionally be called probabilistic. The determining of lineaments and domain boundaries, as well as the additional identification of potential sources, is highly subjective, thus, the location of epicenters and recurrence rate in the model are significantly predetermined by this human factor. The division of seismic zones into linear and areal structures in the LDF model historically appeared as an addition to the deterministic model, which included only linear structures - active faults, and was first proposed by S. A. Cornell [Cornell, 1968]. Such approach was justified in the era of limited computing capacities, when analytical solutions could greatly reduce the amount of calculations. The analytical solutions of S. A. Cornell, thanks to the assumption of spatial homogeneity of earthquake sources within each structure, made it possible to relatively quickly calculate the recurrence of impacts at a given point from potential earthquakes with sources in lineament and domains.

Nowadays, it is no longer necessary to use Cornell's analytical solutions. The PES zone model can be represented in the form of a synthetic catalog of earthquakes. Then, direct calculations at a given point of the impacts from each synthetic earthquake and, accordingly, the rate of impacts of a certain intensity are possible. Of course, it is possible to create a synthetic catalog based on an LDF-type model, but this will not eliminate subjective factors in determining the boundaries of the structural elements of the LDF model that significantly affect the modeling results. Therefore, frequent disagreements among researchers regarding the boundaries of linear and areal structures, criticism of the "characteristic earthquakes" model underlying the identification of individual potential sources have led in many countries to the use of smoothed seismicity – a representation of a seismic regime in which no zone boundaries are needed [Akinci *et al.*, 2018; Helmstetter and Werner, 2012]. The smoothed seismicity approach models the variability between the spatiotemporal distribution of past and future seismicity and provides a much more spatially accurate prediction than zoning models.

Smoothed seismicity models require a significantly larger number of earthquake observations for a detailed and accurate assessment of seismic regime parameters. A quarter of a century has passed since the creation of the GSZ-97 maps, and during this period a huge amount of data on earthquakes has been accumulated, making it possible to move from zoning models to smoothed seismicity models.

The synthetic catalog is formed based on a seismic regime model that determines the recurrence of earthquakes of a certain magnitude at different points in the region under consideration. A common practice is to use recorded earthquakes. It is assumed that the locations of smaller earthquakes are representative of the locations of larger earthquakes. In this case, to estimate the frequency of earthquakes of greater magnitude, the Gutenberg-Richter law is used [Gutenberg and Richter, 1945], which establishes the ratio of the number of events of different magnitudes. The specific details of which magnitudes are used depend on the quality of the earthquake catalog and may change over time. It is generally believed that the reliability of a forecast is related to the length of the catalog, which can vary from decades to centuries.

The choice of smoothing method can be based solely on expert judgment or can be decided on the basis of statistical optimization of forecast accuracy using past earthquakes (e.g. [Petersen *et al.*, 2015; Stirling *et al.*, 2012]). Common practice includes fixed smoothing radius methods (for example, [Frankel, 1995]), and so-called adaptive methods, in which the smoothing radius increases as the density of observed earthquakes decreases [Akinci

et al., 2018; *Helmstetter and Werner*, 2012; *Pisarenko and Pisarenko*, 2021; *Stock*, 2002]. In both approaches, the value of the estimated parameter is assigned to the center of the circle. This always leads to a significant decrease in the spatial resolution of the assessment results: in the most seismically active areas, the activity inevitably turns out to be underestimated, and at the edges of these areas – overestimated. An additional underestimate of activity is caused by the fact that during normalization per unit area, the fractal structure of the seismicity distribution is not taken into account [*Kosobokov and Mazhkenov*, 1992].

Adaptive estimates provide a more contrasting picture of the distribution of seismicity compared to the constant radius methods, however, excessive detail may not be entirely justified due to the significant spatiotemporal clustering of earthquakes, which persists even after declustering of the earthquake catalog. Another drawback of this model is that the assessment of the intensity of the flow of events per time unit and per area unit is based on the assumption of homogeneity, although the method is aimed specifically at identifying the details of the spatial heterogeneity of the seismicity distribution.

Both constant radius and adaptive estimates must take into account the limits of applicability of the Gutenberg-Richter law [*Gutenberg and Richter*, 1945]. The geological basis of this law is the fractal structure of the support of seismicity – the fault system [*Sadovsky*, 1979]. Therefore, when assessing the parameters of the GR law, it is incorrect to consider narrow areas, since this takes into account events only in the “root” part of the fractal structure and thereby predetermines the excess of stronger events. It is also incorrect to consider compact areas (for example, circles) with a radius comparable to the linear size of the source of the strongest earthquake in the area [*Main*, 2000; *Molchan et al.*, 1997; *Romanowicz*, 1992]. Thus, it is necessary to consider areas of a sufficiently large size, but this can lead to an unjustified “smearing” of seismic activity over space.

In this paper, we propose a high-contrast smoothing method in which estimated values are assigned not to the center of the circle, but to the average position of the earthquake epicenters used in the calculations, and in which areal normalization is performed taking into account fractal property of seismicity.

2. Method

We propose a modified version of the smoothing method with circles of a fixed radius, which gives significantly more contrasting results compared to the standard method and does not contain the disadvantages of the adaptive method described above. The choice of smoothing radius can be determined taking into account the applicability of the Gutenberg-Richter law.

The study region is scanned by circles with a constant radius R . The centers of the circles are located at the nodes of a regular grid with a given step D_{lat} in latitude and D_{lon} in longitude. It is well known that seismicity has a fractal structure in space [*Kosobokov and Mazhkenov*, 1992], which must be taken into consideration when normalizing the intensity of the flow of events. This is taken into account in our method. In each circle under consideration, an estimate $\nu(M)$ of the number of earthquakes with a magnitude $m \geq M$ earthquakes in the spatial cell $D_{\text{lat}} \times D_{\text{lon}}$ is made, normalized to the duration of the catalog

$$\nu(M) = N(M) \frac{S_{\text{cell}}^{d_f}}{S_{\text{circle}}^{d_f}}. \quad (1)$$

Here $N(M)$ is the number of earthquakes with magnitudes $m \geq M$ in a circle, $S_{\text{cell}}^{d_f}$ and $S_{\text{circle}}^{d_f}$ are the areas of the cell and the circle in d_f -dimensional space:

$$S_{\text{circle}}^{d_f} = R^{d_f} \frac{\pi^{\frac{d_f}{2}}}{\Gamma(1 + \frac{d_f}{2})};$$

$$S_{\text{cell}}^{d_f} = D^{d_f} \cos^{\frac{d_f}{2}}(\varphi),$$

where Γ is the gamma function, R is the circle radius, D is the linear size of cell in degrees, φ is the latitude of the center of the circle, d_f – fractal dimension of the spatial distribution of the epicentres.

In the modified method proposed here, we will assign the value not to the center of the circle, but to the average position of the earthquakes in the sample. As a result of this operation, there will be several activity values in some cells $0.1^\circ \times 0.1^\circ$. For each such cell, we will choose a single value of ν corresponding to the maximum activity estimate. In those cells that did not contain a single value, we will restore activity using the built-in “Surface” interpolation procedure of the Generic Mapping Tool (GMT) package [Wessel *et al.*, 2019].

The interpolation method used in the “Surface” function of the GMT package is a modification of the minimum curvature method from [Briggs, 1974]. The main problem of interpolation methods when mapping data is getting unreasonable outliers in the results that do not fit well into the real picture (for example, sharp changes in elevation on a plain in terrain maps, etc.). This is caused by the fact that the values in the intervals between points with known data are not constrained by anything [Grain, 1970]. The so-called methods of mathematical surfaces, in which the solution is sought in the form of a single function for the entire surface, and the coefficients are found by solving equations for each point, seem to suffer most from this effect. The problem can be partially solved by dividing the surfaces into separate parts (piecewise), but this increases the computation time and also causes problems with discontinuities at the boundaries [Grain, 1970].

More advanced from this point of view are the methods of numerical surfaces, which can generally be divided into two groups: methods with weights and methods of finite differences. In the first case, the interpolated value is determined by the weighted average of the surrounding points. The further away the point is, the less its weight. However, this method produces poor results when the data density varies greatly. Areas of high density will be smoothed, while areas of low density will be interpolated rather unevenly [Grain, 1970]. It should be noted that in problems of seismicity smoothing, exactly this situation is realized.

The principle underlying finite difference methods is the assumption that the solution surface satisfies some differential equation. This equation is then approximated by finite differences and solved iteratively. In the simplest case, Laplace's equation can be used:

$$\frac{\delta^2 z}{\delta^2 x} + \frac{\delta^2 z}{\delta^2 y} = 0$$

In this case the value at the point z_{ij} will be determined by four other values at the neighboring points [Grain, 1970].

In [Briggs, 1974], a more complex equation is proposed. Specifically, the equation of displacement of a thin metal plate being bent by forces acting at points such that the displacement at these points is equal to the observed value. In this case, the value at point z_{ij} will be determined by twelve values at the neighboring points. It is also shown that such an equation fulfills the condition of minimizing the squared surface curvature. That is, the integral of the squared curvature is proportional to the total elastic energy of the plate and therefore the solution with the minimum energy corresponds to the surface with the minimum squared curvature [Smith and Wessel, 1990]. That is, thereby, the interpolated

values between points with known data are constrained, and a physical justification for these constraints is provided.

However, as shown in [Smith and Wessel, 1990], even such a method can still produce unreasonable fluctuations in surface curvature after interpolation. The authors found out that if horizontal forces are added to the plate equation from [Briggs, 1974], which were excluded there to simplify the calculations, unwanted outliers and unnecessary inflection points can be avoided by controlling the tension parameter.

The method from [Smith and Wessel, 1990], implemented in the “Surface” procedure, has become widespread in solving problems of interpolation of geographical, geological, and geophysical data.

3. Application examples

The figure shows a map for assessing variations in seismic activity in the regions of southern Siberia using the described method. The value $R = 100$ km was used, which is about 10 times the size of the earthquake source with magnitude $M = 6.0$. The map is based on the data from the earthquake catalog for 1982–2021, obtained by combining catalogs from the yearbooks “Earthquakes in the USSR” for 1982–1991 (digitized version is available on the link http://www.wdcb.ru/sep/seismology/cat_USSR.ru.html), yearbooks “Earthquakes in Northern Eurasia” (1992–2021) (http://zeus.wdcb.ru/wdcb/sep/hp/seismology.ru/cat_N_Eurasia.ru.html) and the catalog of the International Seismological Center ISC (<http://www.isc.ac.uk>). Duplicates in the catalog were identified and removed according to the methodology of [Gvishiani et al., 2022; Vorobieva et al., 2022]. The catalog was also declustered using the methodology of [Shebalin et al., 2020]. The resulting catalog is available on the link <https://clck.ru/39T8k6>. Local estimates of magnitude of completeness for the catalog according to the methodology of [Vorobieva et al., 2013] (map is available at <https://clck.ru/39T8oy>) allow us to accept the value $M_c = 3.5$ as a regional estimate. The study region is scanned using circles with a constant radius $R = 100$ km (diameter $D = 200$ km). Value $a = \log_{10} \nu(3.5)$, where $\nu(3.5)$ is an estimate of the number of earthquakes in a cell with magnitude $M \geq 3.5$, calculated using the formula (1). Two versions of estimates can be compared: 1) values are assigned to the centers of the circles, and then interpolation is not carried out, 2) values are assigned to the average position of the epicenters and interpolation is carried out using the method described above. The results for the first option are shown in Figure 1, for the second one – in Figure 2.

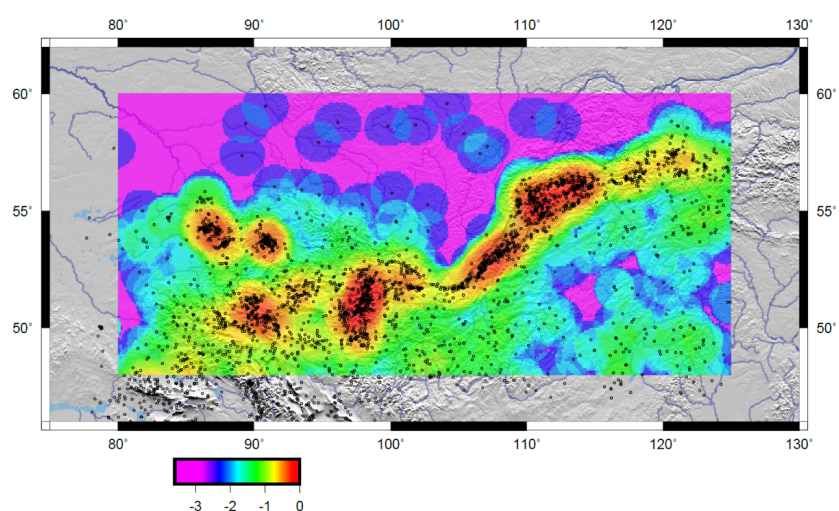


Figure 1. Variations in seismic activity $a = \log_{10} \nu(3.5)$, ν is the estimate of the number of earthquakes with magnitude $M \geq 3.5$, calculated using the formula (1). Earthquake epicenters are shown as black dots. The values are assigned to the centers of the scanning circles.

As it can be seen from Figure 2, when using the second approach, the activity corresponds well to the spatial distribution of the epicenters. If we use the traditional option,

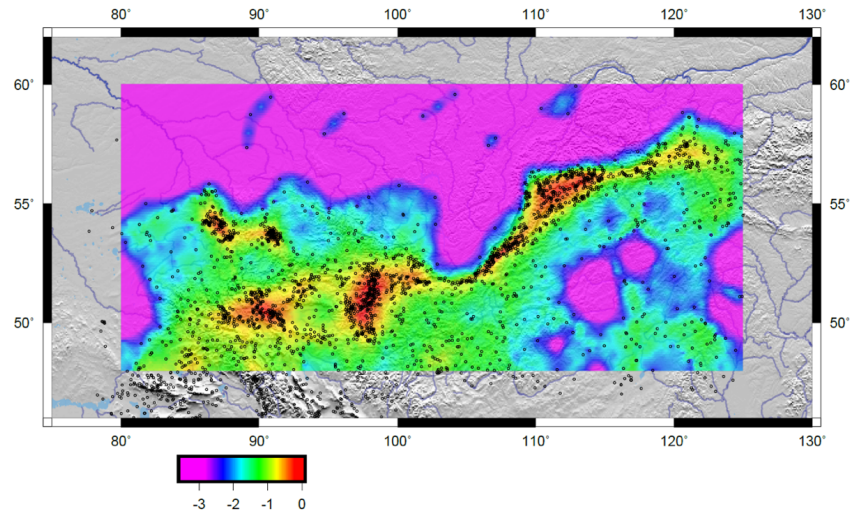


Figure 2. Variations in seismic activity $a = \log_{10} \nu(3.5)$. Earthquake epicenters are shown as black dots. The values are assigned to the average position of the epicenters in the scanning circles. The values at the grid points are calculated using interpolation.

assigning values to the centers of the circles, then the areas of high activity turn out to be significantly “smeared”. Thus, the proposed method, while retaining the advantages of the smoothing method with circles of constant radius, provides a significantly more contrasting activity map, reflecting in detail the distribution of earthquake epicenters.

We propose to use a similar approach for local estimates of the b -value parameter. Since earthquakes of magnitude 7 have been observed in the region, to estimate the b -value parameter we use an increased value of $R = 200$ km, which corresponds to the conditions for the applicability of the Gutenberg-Richter law. The b -value is calculated using the Bender method [Bender, 1983], which, unlike the widely used Aki method [Aki, 1965], gives an unbiased value on small samples. The minimum sample size is 50 events; if the number of events in the circle is less than 50, we assign the corresponding cell the regional value of b .

Similar to the activity maps (Figure 1 and Figure 2), we compare two options for mapping the b -value: 1) values are assigned to the centers of circles (Figure 3), 2) values are assigned to the average position of the epicenters with subsequent interpolation (Figure 4).

As it can be seen from the figures, the use of our method actually leads to a significantly more contrasting picture of the distribution of the b -value: the areas of anomalously large and anomalously small values of the parameter turn out to have much smaller area. At the same time, unlike the activity maps, areas of homogeneous parameter values show low correlation with the density of epicenters and do not have an elongated shape.

4. Discussion and conclusion

In this paper, we proposed a high-contrast smoothing method for local estimation and mapping of seismic regime parameters determined by the Gutenberg-Richter law. When using this method, on the one hand, the conditions for the applicability of this law are controlled (a fairly large smoothing radius), and on the other hand, narrow areas of really high seismic activity are not “smeared” into wider zones that are not actually active at the edges.

The proposed approach allows to obtain local estimates of the parameters of the Gutenberg-Richter law with arbitrary detail. We used a scanning step of 0.1 degrees by latitude and longitude. For comparison, the detail of estimates when constructing GSZ-97, GSZ-2015, GSZ-2016 was 0.25 degrees. As the analysis of the obtained results shows, there is no need to further increase the detail, since the obtained estimates in neighboring grid cells differ, as a rule, by no more than 30%. If necessary, a technical increase in detail can be achieved using linear interpolation.

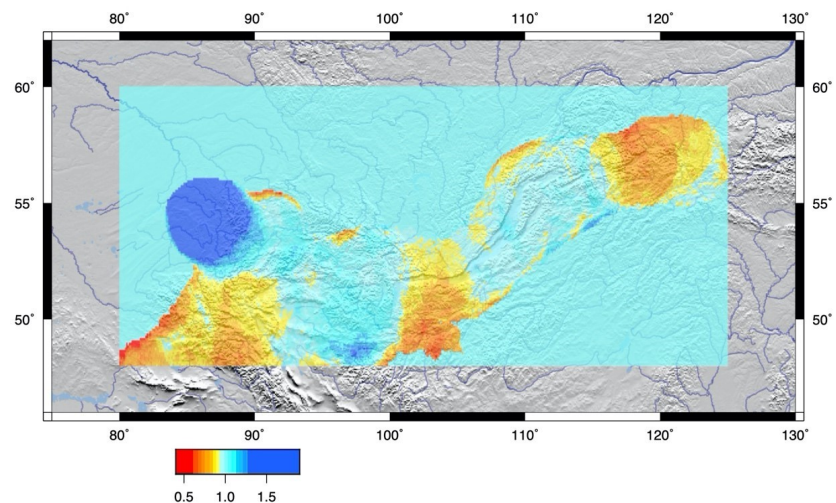


Figure 3. Variations of the b -value parameter. The values are assigned to the centers of the scanning circles.

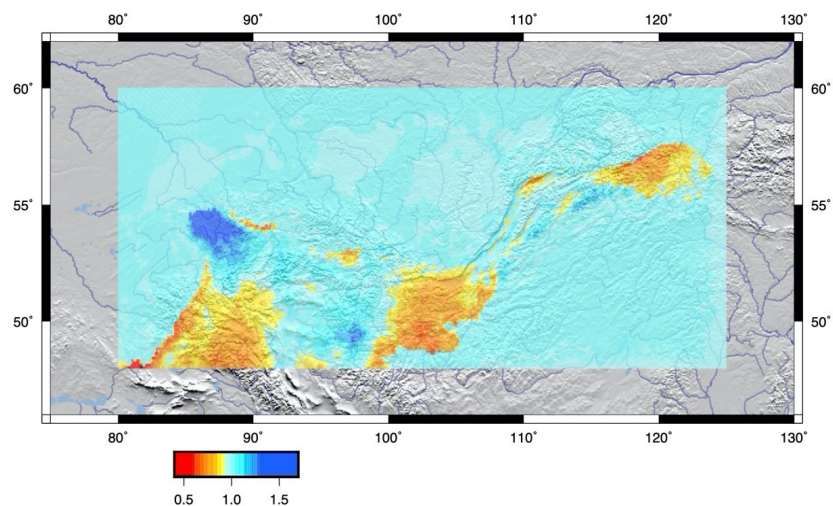


Figure 4. Variations of the b -value parameter. The values are assigned to the average position of the epicenters in the scanning circles. The values at the grid points are calculated using interpolation.

The local estimates of the parameters of the Gutenberg-Richter law obtained using our method can be used to construct a synthetic catalog of earthquakes. To do this, it is necessary to estimate the parameters of the Gutenberg-Richter law for the entire simulated region. This will allow to generate a sequence of seismic events with determined times (based on the Poisson model) and magnitudes. To generate the coordinates of the epicenter, it is enough to use local estimates of a and b -value to construct a probability distribution for the occurrence of an event of a given magnitude in cells of a regular grid and, using this distribution, select the appropriate cell using a random number generator. For the epicenters of the strongest earthquakes, additional constrains can be introduced using some model of the places where strong earthquakes may occur. Modeling of background earthquakes that follow the Poisson distribution can be supplemented with a synthetic catalog of clustered events, generated as sequences of aftershocks from background events, aftershocks of aftershocks, etc.

The approach to constructing a synthetic catalog described above does not require any subjective decisions other than choosing the boundaries of the region. This is its main advantage compared to LDF-type models. However, like the LDF model, this is just one of the possible solutions. To evaluate which approach is more effective, a quantitative

comparison of different models with a catalog of recorded earthquake is necessary. The methodological foundation for such a comparison can be tests based on the likelihood function [Zechar *et al.*, 2010]. Comparison with the catalog that was used to build the model will only indicate the internal compatibility of the method and data. To compare the efficiency of different models a test on independent data is necessary. One can, for example, consider strong earthquakes for the period preceding the start date of the used part of the catalog.

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