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The mathematical model of thermoelastic stresses in the Carpathian region

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The two-dimensional thermoelastic model of the Carpathian region is constructed using the finite element method. The physical-mathematical model is as close as possible to the real environment. A high-precision detailed distribution of temperatures, heat flow, thermoelastic stresses and displacements is obtained. It is shown that inhomogeneous heating of rocks and structural heterogeneities lead to significant changes in stress and displacement fields in the Carpathian region.

Keywords: temperature, heat flow, thermoelastic stresses, displacements, inhomogeneous medium.

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1 Introduction

Numerous theoretical and experimental geophysical studies show that the temperature changes observed in the Earth's crust create thermoelastic stresses, the values of which reach the strength limit of rocks. This gives us reason to consider thermal stresses as one of the sources of tectonic and seismic activity.

The analysis of the heat field features of the Carpathians and adjacent troughs shows that the heat flow distribution correlates with the features of the Carpathian region tectonics [*Kutas*, 2014].

Major structural elements of the Carpathians are the Transcarpathian trough, the Folded Carpathians and the Pre-Carpathian trough. Each of these structural elements is distinguished by the history of its development, the age and structure of the foundation, the sedimentation conditions, the Earth's crust thickness [*Glushko*, 1968; *Chekunov*, 1987].

The Carpathians folded-cover structure indicates significant horizontal movements of the lithosphere plates.

The heat flows value in the Folded Carpathians reaches 50–75 $\mathrm{mWm^{-2}}$, in the Flysch Carpathians it does not exceed 65 $\mathrm{mWm^{-2}}$. The distortions of heat flows associated with the cover structure of the Carpathian system is 10–15% [*Kutas*, 2014].

The Outer Folded Carpathians and the Transcarpathian trough are separated by the Transcarpathian deep fault. In the Transcarpathian trough heat flows increase up to 80–90 $\mathrm{mWm^{-2}}$, and within the anomalous zones to 100–115 $\mathrm{mWm^{-2}}$ [*Kutas*, 2014].

The mountain-folded structure of the Carpathians is separated from the Pre-Carpathian trough by the deep Pre-Carpathian fault. A high heat flows anomaly is observed in the northwestern part of the Pre-Carpathian trough and along its southwestern side (50–70 $\mathrm{mWm^{-2}}$). In the lowest zone, where the sediment capacity reaches 8–10 km, minimal heat flows are observed [*Kutas*, 2014].

Seismic studies of the Carpathian region clearly indicate that the seismically active areas correspond to increased values of heat flow. In Transcarpathia, the magnitude of seismic events is within $M_{max} = 4.7 \div 5.3$, and the heat flow density $q = 80 \div 100$ $\mathrm{mWm^{-2}}$. In the Carpathians $M_{max} = 4.2 \div 4.7$, $q = 60 \div 70$ $\mathrm{mWm^{-2}}$. In the northeast of the Pre-Carpathian trough $M_{max} = 4.7$, $q = 50$ $\mathrm{mWm^{-2}}$; in the southeast – $M_{max} = 4.1$, $q = 40$ $\mathrm{mWm^{-2}}$ [*Kutas et al.*, 2006].

The tectonic fragmentation of the Earth's crust, the presence of contrasting inclusions and structural heterogeneities cause significant variations in the heat flow both laterally and in depth. Thermal anomalies associated with contact zones are quite intense and occupy vast territories [*Bakhova*, 2009].

Inhomogeneous heating of a heterogeneous environment leads to the occurrence of thermoelastic stresses and displacements. The evaluation of the purely thermoelastic contribution to the lithosphere stress state shows that thermal stresses can have a significant influence on the formation of seismically active zones [*Bakhova*, 2010].

Further study of thermoelastic effects is of great interest, since all earthquake mechanisms are directly or indirectly related to the Earth's thermal field. In this paper, the method for calculating

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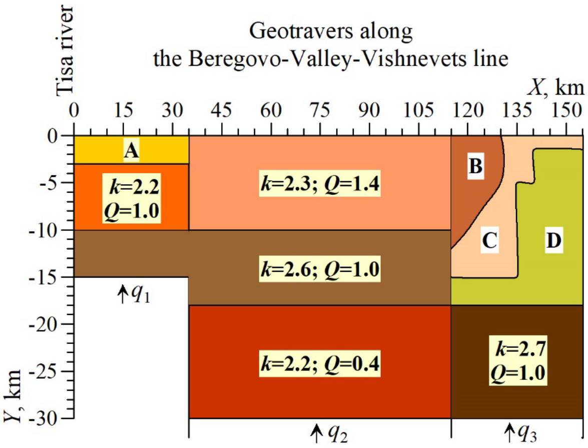

Figure 1: Geothermal model of the Carpathian region $A - k = 2.0\,\mathrm{Wm^{-1}K^{-1}}$; $Q = 1.2\,\mu Wm^{-3}$; $B - k = 2.0\,Wm^{-1}K^{-1}$; $Q = 1.4\,\mu Wm^{-3}$; $C - k = 1.7\,Wm^{-1}K^{-1}$; $Q = 1.4\,\mu Wm^{-3}$; $D - k = 2.4\,Wm^{-1}K^{-1}$; $Q = 0.8\,\mu Wm^{-3}$.

thermoelastic stresses and displacements in the Carpathian region based on the numerical finite element method is proposed.

2 Problem statement and solution methods

The fundamental point in solving the thermoelastic problem is the determination of temperature changes ΔT.

It is known from the thermoelasticity theory that temperature changes ΔT are counted from the value T_0, in which the body state is considered undeformed in the absence of external forces [*Landau and Lifshitz*, 1987].

In geophysics, it is quite difficult to determine the undeformed state temperature of a geological body. It is necessary to accept several conditional agreements.

There are two stages in the Carpathians geothermal activity.

The first stage coincides with the initial formation stage of the Carpathian geosyncline. The regional anomaly of weak intensity is caused by a heat source formed in the upper mantle 160–180 million years ago.

The main source of thermal energy at the second stage was located inside the Carpathian region, west of the Transcarpathian deep fault. This activation stage took place about 25–30 million years ago and was caused by the asthenosphere rise and the lithosphere restructuring [*Kutas*, 2014].

Taking into account the history of thermal and geological development of the Carpathian region, the first model approximation is formed.

To determine the value T_0 the absence of mantle heat sources under the Carpathians and the Transcarpathian trough is accepted, and homogeneous heating of the studied region is assumed. In addition, deformations caused by homogeneous heating of a homogeneous environment are considered negligible. In other words, the temperature T_0 for each node of the finite element grid was calculated at the mantle heat flow $q_1 = 25\,\mathrm{mWm^{-2}}$, $q_2 = q_3 = 15\,\mathrm{mWm^{-2}}$ (Figure 1).

The thermal regime analysis of the Carpathian region is based on geological and geothermal data along the geotraverse II, crossing the Transcarpathian trough, the Carpathians, the Pre-Carpathian trough and the East European Platform [*Chekunov*, 1987].

For the upper layers of the Carpathian region geothermal model, thermal conductivity of rocks k and the radiogenic heat generation Q are given based on the experimental data basis [*Kutas*, 2014].

On the upper service of the model the boundary Dirichlet condition for the Transcarpathian trough is $T = 12°\mathrm{C}$, for the Carpathians is $T = 6°\mathrm{C}$ and in the area of the Pre-Carpathian trough is $T = 9°\mathrm{C}$. The mantle component values of the heat flow are determined using a computational experiment (Figure 1): $q_1 = 70\,\mathrm{mWm^{-2}}$; $q_2 = 30\,\mathrm{mWm^{-2}}$; $q_3 = 15\,\mathrm{mWm^{-2}}$.

3 The results obtained and their discussion

The calculation results of temperatures and surface heat flow are shown in Figure 2.

Data about the temperature distribution in the region are the initial information for calculating thermoelastic stresses and displacements.

When solving the problems of thermal conductivity and thermoelasticity joint finite element grids were used.

For a plane stress state in an element of an isotropic material that is subject to a temperature change by ΔT with a thermal expansion coefficient α the following relation can be written

$$\varepsilon_0 = \begin{bmatrix} \alpha \Delta T^e \\ \alpha \Delta T^e \\ 0 \end{bmatrix}, \tag{1}$$

since there is no shear deformation during thermal expansion (compression) [*Zienkiewicz and Cheung*, 1971].

Stresses and deformations are related to each other by the following relation

$$\sigma = D(\varepsilon - \varepsilon_0) \tag{2}$$

where D – elasticity matrix reflecting the material properties (Young's modulus and Poisson's ratio).

Equation (2) is valid when the lithosphere rocks behave elastically for millions of years. Thus, to determine the maximum temperatures at which

(2) is true, it is necessary to find the elastic lithosphere thickness.

In the geophysical literature, the temperature of the elastic layer base is called the elastic "blocking temperature" T_e [*Bratt et al.*, 1985; *McNutt and Menard*, 1982].

The lithosphere rocks, heated to the certain temperature $T > T_e$, cannot make a significant contribution to the field of thermal stresses, since the stresses will be released plastically.

However, there are other estimates T_e.

In the works [*McNutt and Menard*, 1982; *Bodine et al.*, 1981] there is an obvious correlation between the elastic thickness of the oceanic lithosphere and the depth 450°C–isotherm calculated from the cooling model of the spreading oceanic plate. It is impossible to apply thermal and thermomechanical models of the oceanic lithosphere to the continental one [*Willett et al.*, 1985].

The correlation observed in the oceanic lithosphere between the heat flow and the age of structures, also between the thickness of the elastic-viscous layer and the characteristic temperatures on the continents, is more complex. Authors [*Willett et al.*, 1985] concluded that the elastic power corresponds to depths with different isotherms: from 300°C before 700°C.

Limit value T_e for the oceanic lithosphere is 700–800°C [*Bratt et al.*, 1985].

The temperature in the earthquake source area is an important factor in determining whether the deformation occurs seismically or not. The maximum temperatures at which earthquakes occur in the Earth's crust are within 350 ± 100°C, and in the mantle – 700 ± 100°C [*Chen and Molnar*, 1983]. Most near-horizon earthquakes have focal depths between the Moho and the depth 800°C isotherm [*Bratt et al.*, 1985].

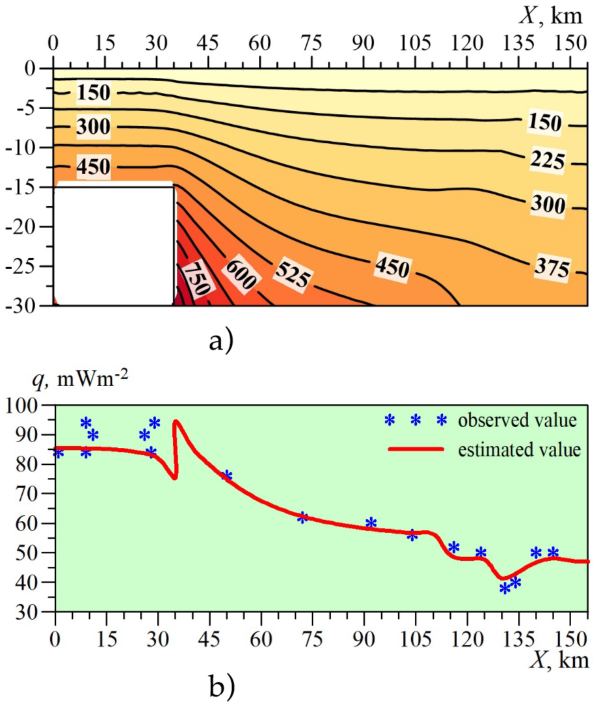

Figure 2: Thermal model of the Carpathian region: a) – temperature distribution; b) – surface heat flow distribution.

In the Transcarpathian trough the magnitude of the strongest earthquakes corresponds to 4.7–5.3, the depth of their focus does not exceed 10 km [*Kutas et al.*, 2006]. The temperature at these horizons reaches 500°C (Figure 2, a).

The second approximation of the Carpathian region thermoelastic model is as follows. The environment is considered to be a conductor of thermoelastic stresses to the depth of 500°C isotherm. It is from these considerations that the model lower bound is chosen.

In the areas of transition from the Transcarpathian trough to the Carpathians at depths from 15 km to 30 km the temperature reaches 800°C. This zone is included in the model, but the thermoelastic stresses accumulated can be released plastically.

For two-dimensional problems of solid mechanics, based on the minimum potential energy principle, the following functional can be obtained

$$\Pi = \int_V \frac{1}{2}\varepsilon^T \sigma \, dV. \tag{3}$$

For a finite element partitioning, expression (3) can be written as

$$\Pi^e = \sum_{e=1}^{l} t_e \int_{\Omega_e} \frac{1}{2}\varepsilon^T \sigma \, d\Omega_e,$$

where l – total number of elements in the system;

t – element subdomain thickness;

Ω_e – element subdomain.

Differentiating an element contribution Π^e by displacement the grid nodes δ and by summing over all the elements, we get a system of algebraic equations

$$K\delta = F_0, \tag{4}$$

where K – stiffness matrix;

F_0 – forces in the nodes that occur during the initial deformation.

Equation (4) allows us to obtain a solution for displacements. From the relations (1) and (2), we can obtain the following expression for the stresses

$$\sigma = DB\delta - D\varepsilon_0. \tag{5}$$

It is supposed that the stresses inside the elements are constant and are applied to the element gravity center. The matrix B is defined if the shape functions and nodal displacements are known.

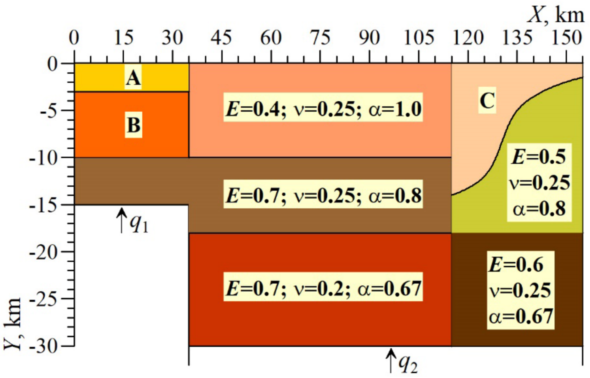

Figure 3: Thermoelastic model of the Carpathian region: A – $E = 0.2$, $\nu = 0.2$, $\alpha = 1.0$; B – $E = 0.4$, $\nu = 0.25$, $\alpha = 1.0$; C – $E = 0.3$, $\nu = 0.2$, $\alpha = 1.0$.

Environment elastic properties (Young's modulus $E \times 10^{11}$ Pa, Poisson's ratio ν, coefficient of linear expansion $\alpha \times 10^{-5}$/°C), shown in Figure 3, set based on generalized data [*Turcotte and Schubert*, 1982].

Without introducing the minimum number of known displacements solution to system (4) is impossible, because the displacements cannot be unique way determined through forces. Mathematically this obvious fact means that the matrix [**K**] becomes singular, that is invertible.

The choice of known displacements or model "fixing" depends on the specific geodynamic situation.

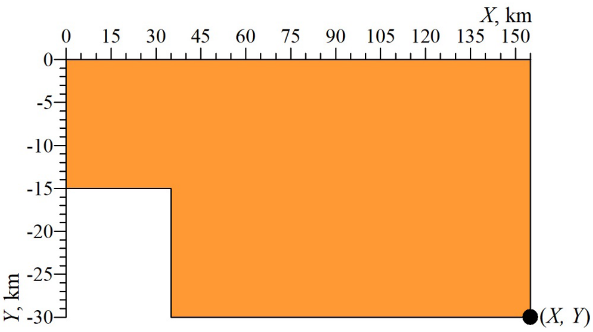

Figure 4: Setting zero displacements: the model is fixed at one point in the axis directions X and Y.

Setting the known zero displacements in the Carpathian region is somewhat difficult. In such situations, a computational experiment comes to the rescue. The calculations were carried out for the "fixing" model shown in Figure 4.

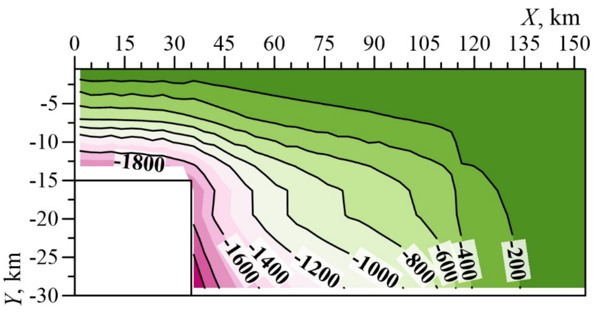

Figure 5: Distribution of thermoelastic stresses σ_{yy}.

The calculation results of thermoelastic stresses σ_{yy} are presented in Figure 5. The stresses σ_{xx} have a similar distribution pattern. The σ_{yy} and σ_{xx} are stresses of the tensile type and are the same, since the model "fixing" does not affect the values of thermoelastic stresses. This follows from the formula (5).

The maximum values σ_{yy} (1.5–2 kbar) are located in the region of the Transcarpathian trough and the Folded Carpathians. The Eastern European platform is characterized by low stresses values (5–240 bar).

The displacement of environment nodal points for the model are shown in Figure 6. The "minus" sign in the axis X direction means that the environment is moving in the direction opposite to the axis X (Figure 6, a).

In turn, the "minus" sign in the axis Y direction means that the environment's movement occurs in the axis Y direction. The values of the movements are given in meters.

The Earth's crust of the studied region is experiencing stretching, the maximum value of which is observed in the Transcarpathian trough.

The environment lowering occurs in the transition area from the Transcarpathian trough and directly under the Carpathians (Figure 6, b).

Towards the East European Platform, the Earth's crust vertical movements gradually decrease.

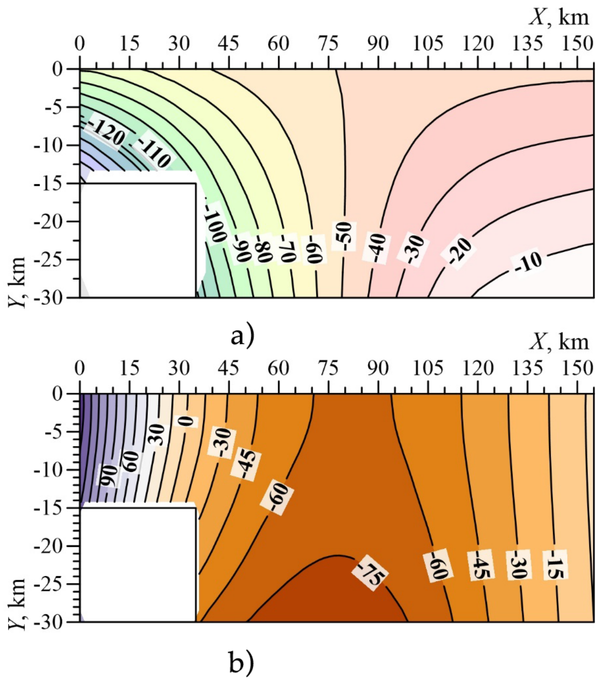

Figure 6: Distribution of displacements: a) – displacements in the axis X direction; b) – displacements in the axis Y direction.

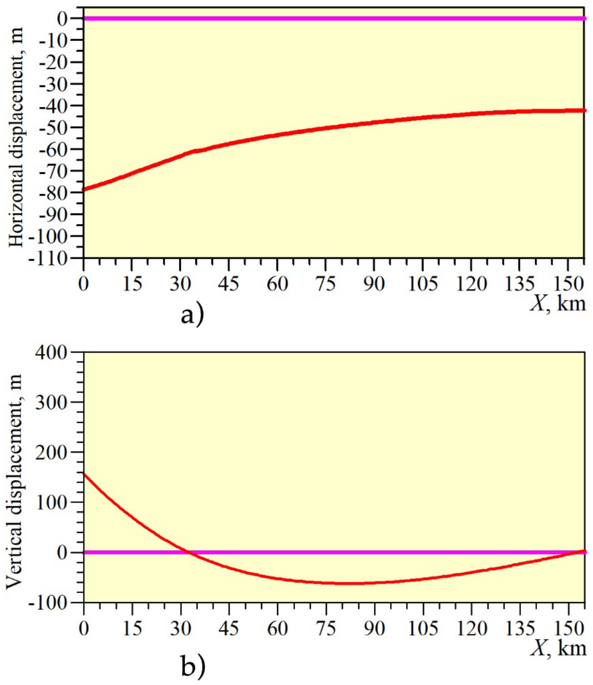

Figure 7: The Earth's crust surface displacement for model: a) – horizontal displacement; b)– vertical displacement.

Figure 7 shows the horizontal and vertical movements of the Earth's surface.

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