Global View on Statistical Models of Sea Surface Elevations

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Abstract: The verification of statistical models of sea surface elevations based on the decomposition of the wave profile into degrees of a small parameter (wave steepness) and in terms of multidimensional integrals of wave spectra was carried out. For verification, wave measurement data were used to calculate the skewness and excess kurtosis of surface elevations, as well as the distribution of crests and troughs. Two factors are identified that limit the use of estimates of skewness A_η and excess kurtosis E_η obtained from existing models. First, the model estimates A_η and E_η are always non-negative, although the measurement data show that the lower limit of the ranges in which the skewness and excess kurtosis change is in the region of negative values. Secondly, almost all existing models are one-parameter models, using wave steepness and wave age as predictors; whereas the measured data indicate that there is no clear relationship. The values of A_η and E_η vary greatly for fixed values of the predictors. Existing statistical models can only describe average changes A_η and E_η. This limits the scope of their application. The analysis of the probability density functions of the troughs F_{Th} and crests F_{Cr} showed that the function calculated for $A_\eta < 0$ in the region above the distribution mode exceeds the values corresponding to the Rayleigh distribution, and the relationship $F_{Th} \approx F_{Cr}$ holds. The second order nonlinear model is inconsistent with this result. Negative skewness values are observed much less frequently than positive ones, so the functions F_{Th} and F_{Cr} calculated for the whole ensemble of situations are consistent with the second-order nonlinear model.

Keywords: sea surface, statistical models, skewness, excess kurtosis, crest distributions, trough distributions.

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Introduction

Despite the fact that Stokes published a paper in 1849 [*Stokes*, 1849] showing the kinematic nonlinearity of the profile of finite amplitude surface waves, the linear model remained the dominant model of sea surface waves for a long time. The linear model represents the wave field as the sum of a large number of independent sinusoidal components, the whose amplitudes are random variables, and the whose phases are uniformly distributed over $[0, 2\pi]$. Such a model assumes, according to the central limit theorem, that the elevation and slope of the surface follow a Gaussian distribution [*Longuet-Higgins*, 1957]. Within the linear model, the distribution of wave heights and the distributions of crests and troughs are described by the Rayleigh distribution [*Longuet-Higgins*, 1952; *Naess*, 1985].

At the beginning of the second half of the last century, an active research on the nonlinear effects in the wave field was started. It was found that the nonlinear interaction between the components of the wave field leads to deviations from the Gaussian distribution [*Longuet-Higgins*, 1963; *Phillips*, 1960]. Wave measurements have shown that the distribution of sea surface elevations is better described by the Gram–Charlier distribution than by the Gaussian distribution [*Kinsman*, 1965]. Laboratory experiments have confirmed that the statistical moments of the elevations of the water surface created by the waves deviate from the values corresponding to the Gaussian distribution [*Huang and Long*, 1980]. The theory of modulation instability of Stokes waves was developed [*Benjamin and Feir*, 1967; *Zakharov*, 1967].

The senior statistical moments (or cumulants) of sea surface elevations are indicators of the nonlinearity of sea waves. Several types of models are used to calculate them. The most widely used models based on the Stokes wave [*Huang et al.*, 1983; *Jha and Winterstein*, 2000; *Tayfun and Alkhalidi*, 2016]. As a general rule, these models take into account the second term in the Stokes expansion, that is, they are second order nonlinear models. Models based on the approximate third order Stokes expansion are not commonly used [*Boccotti*, 2000]. In this paper, we will limit ourselves to analyzing the possibility of calculating the senior cumulants of sea surface elevations using a second order nonlinear model. Also in the last two decades, models have been created in which skewness and excess kurtosis are calculated from wave spectra [*Annenkov and Shrira*, 2013, 2014; *Janssen*, 2003; *Janssen and Bidlot*, 2009; *Mori and Janssen*, 2006].

Waves on the sea surface are always random in the sense that the topography of the surface changes in an irregular manner in both time and space. The main criteria used in the verification of statistical models of sea surface elevation distributions is the correspondence of the statistical moments calculated within their framework to the data of direct wave measurements. An additional criterion is the deviation of the distributions of heights, crests and troughs from the Rayleigh distribution. To verify statistical models, this paper summarizes the results of experiments conducted at the Marine Hydrophysical Institute of the Russian Academy of Sciences on a stationary oceanographic platform located in the Black Sea [*Zapevalov*, 2024; *Zapevalov and Garmashov*, 2021, 2022, 2024].

Statistical Description

This section defines the main characteristics of waves that are used for statistical analysis of non-Gaussian sea states. Here we are going to use the useful statistical moments, which can be calculated from measuring the surface elevations at a fixed point, as follows

$$\mu_n = \left\langle \eta^n(t) \right\rangle$$

where $\eta(t)$ is the surface elevation, t is the time, the angular brackets mean averaging. It is assumed that the mean value of a random variable η is zero $(\mu_1 = 0)$. To analyze the effects associated with wave nonlinearity, skewness and excess kurtosis are commonly us $A_\eta = \mu_3/\mu_2^{3/2}$ and $E_\eta = \mu_4/\mu_2^2 - 3$. The energy of the wave field is determined by the second statistical moment. Usually in oceanography, instead of μ_2, the significant wave height H_S is used, equal to the average height of 1/3 of the highest waves, which is determined by the ratio

$$H_S = 4\sqrt{\mu_2}. \quad (1)$$

Given (1), the wave steepness can be defined as $\varepsilon = \frac{H_S}{4}k_p$, where k_p is the wave number of the peak of the wave spectrum. The inverse wave age is defined as $\zeta = \frac{U_{10}}{C_p}$, where U_{10} is the wind speed at an altitude of 10 m; C_p is the phase velocity of a wave with a wave number k_p [*Donelan et al.*, 1985; *Young and Donelan*, 2018]. The smaller the value of ζ, the later the stage of development of the waves corresponds to. The value $\zeta_0 = 0.83$ corresponds to a fully developed wave, at $\zeta < \zeta_0$ the wave field will create a swell, at $\zeta > \zeta_0$ wind waves will be created.

The changes in the wave steepness at different stages of the wave field development are shown in Figure 1. For the swell, the correlation coefficient between the parameters ζ and ε is 0.15, the dependence of the wave steepness on the stage of development is approximated by linear regression [*Zapevalov and Garmashov*, 2021] $\varepsilon = 0.016 + 0.0061\zeta$. For wind waves, the correlation between these parameters is much higher and equal to 0.65, the linear regression equation has the form $\varepsilon = 0.021\zeta$.

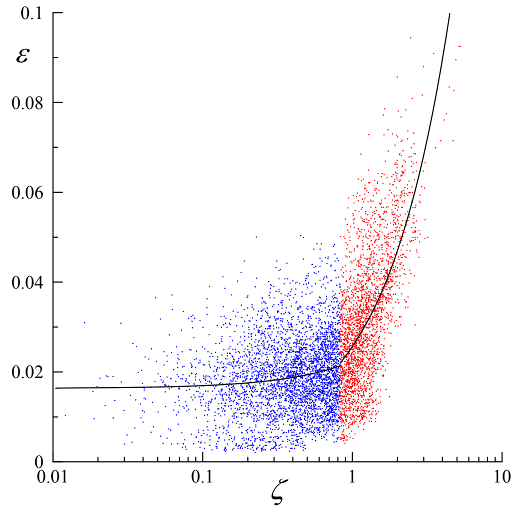

Figure 1. The correlation between the inverse wave age ζ and wave steepness ε.

A Second-Order Nonlinear Model

The analysis of nonlinear effects is usually carried out within the framework of models that are based on the decomposition of the wave profile by degrees of a small parameter ε. The steepness of the sea waves lies within the limit $0 < \varepsilon < 0.1$ [*Zapevalov and Garmashov*, 2021]. The profile of the wave is represented as the sum of the linear and nonlinear components $\eta(x,t) = \eta_L(x,t) + \eta_N(x,t)$. The linear component is a superposition of sinusoidal waves $\eta_L(x,t) = \sum_{n=1}^{\infty} a_n \cos\psi_n$, where x is the spatial coordinate, a_n are amplitudes of the wave components, $\psi_n = k_n x - \omega_n t + \varphi_n$, k_n and ω_n are a wave number and an angular frequency, φ_n is the phase. The amplitudes are Rayleigh distributed random variables. In a second order nonlinear model, terms proportional to ε are considered. In recent years, a model has been widely used in which the nonlinear component is given in the form [*Gao et al.*, 2020; *Jha and Winterstein*, 2000; *Toffoli et al.*, 2007].

$$\eta_2(x,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{a_m a_n\left[B_{mn}^{-}\cos\left(\psi_m - \psi_n\right) + B_{mn}^{+}\cos\left(\psi_m + \psi_n\right)\right]\right\},$$

where transfer functions B_{mn}^{-} and B_{mn}^{+} are obtained from solution of Laplace's equation for the velocity potential with nonlinear boundary conditions.

Relationships linking skewness and excess kurtosis to wave steepness are obtained in [*Tayfun and Alkhalidi*, 2016] for a second-order nonlinear model

$$A_\eta = 3\varepsilon + O\left(\varepsilon^3\right), \tag{2}$$

$$E_\eta = 12\varepsilon^2 + O\left(\varepsilon^4\right). \tag{3}$$

Let's compare the model dependence (2) with the results of calculations based on wave measurements (Figure 2). For a swell, the statistical relationship between skewness and wave steepness is approximated by linear regression

$$A_{swell} = 2.52\varepsilon + 0.02. \tag{4}$$

For wind waves (ww), the linear regression has the form

$$A_{ww} = 1.79\varepsilon + 0.05. \tag{5}$$

The values A_η calculated within the framework of nonlinear models, including within the framework of the second-order nonlinear model, are always positive [*Huang et al.*, 1983;

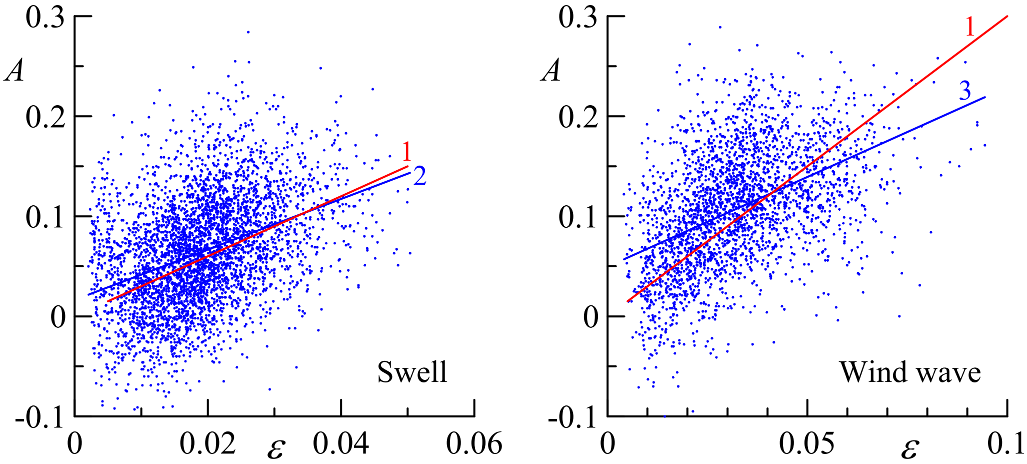

Figure 2. The dependence of asymmetry on steepness. The points are measurement data, lines 1, 2 and 3 correspond to equations (2), (4) and (5).

Naess, 1985; Tayfun and Alkhalidi, 2016]. This is in contrast to field measurements where the lower limit of the range in which skewness values are found is in the region of negative values [Guedes Soares et al., 2004; Jha and Winterstein, 2000; Zapevalov and Garmashov, 2022]. In laboratory experiments, negative skewness values were also observed. [Huang and Long, 1980; Zavadsky et al., 2013]. In addition, the model dependencies provide a clear link between A_η and ε, whereas Figure 2 shows that for fixed values ε the values A_{swell} and A_{ww} vary within wide limits.

Similar contradictions between model calculations and measurement results occur for excess kurtosis and wave steepness. In Figure 3, in addition to the model dependence (3), linear and quadratic regressions are shown. These are described by the equations

$$E_{swell} = 0.93\varepsilon - 0.02, \tag{6}$$

$$E_{swell} = 32.63\varepsilon^2 - 0.43\varepsilon - 0.01, \tag{7}$$

$$E_{ww} = 1.01\varepsilon, \tag{8}$$

$$E_{ww} = -11.63\varepsilon^2 + 1.87\varepsilon - 0.01. \tag{9}$$

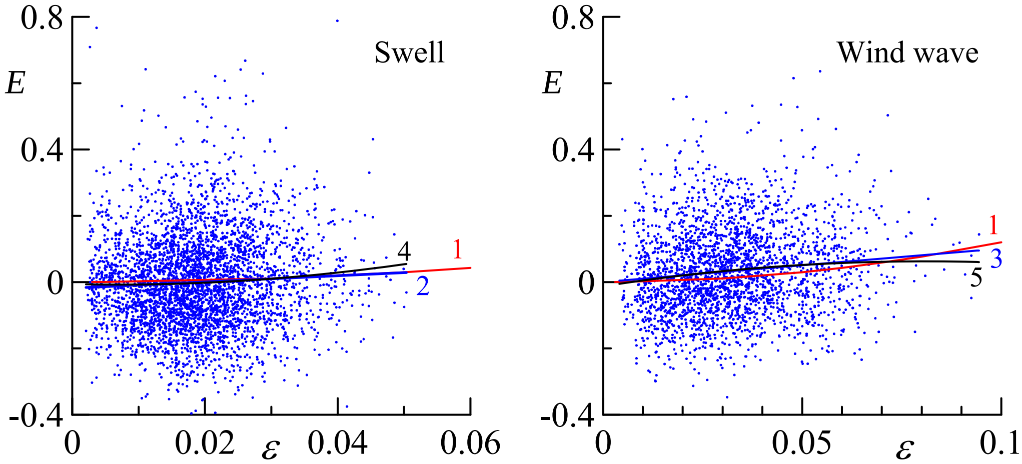

Figure 3. Dependence of the excess kurtosis on the wave steepness. The points are the measurement data, line 1–5 correspond to equations (3), (6), (7), (8) and (9).

Spectral Wave Model

Currently, global and regional spectral wave models have become a working tool for sea wave forecasting [*Grigorieva et al.*, 2020]. Comparison with in situ and remote measurement data shows that these models describe the spatial and temporal changes in wave energy quite well [*Mikhailichenko et al.*, 2016; *Stopa et al.*, 2016]. In [*Janssen*, 2003; *Janssen and Bidlot*, 2009], on the basis of the canonical transformation in the Hamiltonian theory of water waves, relations were obtained linking skewness and excess kurtosis with the characteristics of the wave spectrum. If these relationships are correct, then the results of calculations using spectral wave models can be used to correct the data from altimetric measurements by reducing the sea state bias [*Badulin et al.*, 2021; *Cheng et al.*, 2019]. They can also be used to predict dangerous water areas, since the excess kurtosis is a relevant parameter in the detection of freak waves [*Luxmoore et al.*, 2019; *Pelinovsky and Shurgalina*, 2016].

It has been shown that the skewness calculated in terms of the directional wave spectrum is always positive [*Longuet-Higgins*, 1963]. In [*Mori and Janssen*, 2006], for a narrow-band wave train, the dependence of the skewness and the excess kurtosis on the steepness of the waves is obtained as follows

$$A_\eta = 3\varepsilon, \tag{10}$$

$$E_\eta = 24\varepsilon^2. \tag{11}$$

For the wave spectral model, the dependence of the skewness on the wave steepness coincides with the dependence (2) obtained in the framework of a second-order nonlinear model. In equations (3) and (11) the numerical coefficients for ε^2 differ by a factor of two. The limitations of using equations (10) and (11) were shown in the previous section.

In a special case for the modified JONSWAP spectrum, whose parameters are explicit functions of the age of the waves, the following dependence was obtained [*Annenkov and Shrira*, 2014]

$$E_\eta = 0.04 + 0.082\zeta^{0.87}. \tag{12}$$

The modified JONSWAP spectrum for wind waves satisfying the condition $0.83 < \zeta < 5$ is [*Donelan et al.*, 1985]

$$E_D(\omega,\theta) = 4\pi^2 \frac{\alpha_D g^2}{\omega^5}\left(\frac{\omega}{\omega_p}\right) exp\left[-\left(\frac{\omega}{\omega_p}\right)^2\right] \gamma_D^{\,exp\left[-\frac{\left(\frac{\omega}{\omega_p}-1\right)^2}{\left(2\sigma_D^2\right)}\right]}\Theta(\theta),$$

where θ is the azimuth angle, $\alpha_D = 0.006\zeta^{0.55}$, $\sigma_D = 0.08\left(1+\frac{4}{\zeta^3}\right)$,

$$\gamma_D = \begin{cases} 1.7 & \text{if} \quad 0.83 < \zeta < 1 \\ 1.7 + 6.0 lg\zeta & \text{if} \quad 1 \le \zeta < 5 \end{cases},$$

$\Theta(\theta)$ is a spreading function. The spreading function is defined as $\Theta(\theta) = \frac{1}{2}\beta \mathrm{sech}^2(\beta\theta)$,

$$\text{where } \beta = \begin{cases} 261\left(\frac{\omega}{\omega_p}\right)^{1.3} & \text{if} \quad 0.56 \le \frac{\omega}{\omega_p} < 0.95 \\ 228\left(\frac{\omega}{\omega_p}\right)^{-1.3} & \text{if} \quad 0.95 \le \frac{\omega}{\omega_p} < 1.6 \\ 1.24 & \text{if} \quad 1.6 < \frac{\omega}{\omega_p} \end{cases}.$$

The agreement of formula (12) with measurement data is shown in Figure 4. The same figure shows a linear regression relationship, which looks like this:

$$E_\eta = 0.029\zeta.$$

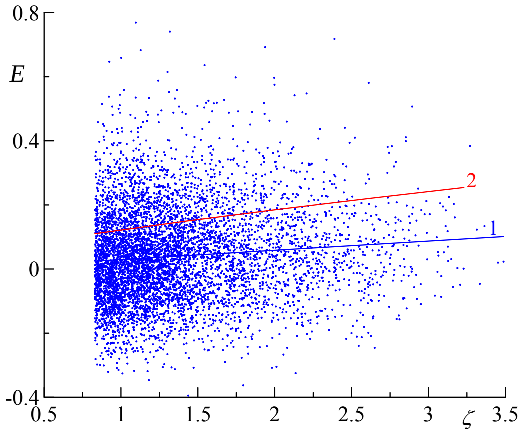

Figure 4. Dependence of the excess kurtosis on the inverse wave age. The points are measurement data, the line 1 is a dependence (12).

Distribution of Crests and Troughs

The deviation of the wave height distribution due to nonlinearity is small for the main part of the distribution, but not for the tail, and is therefore of great importance in predicting the occurrence of abnormally high waves [*Stansell*, 2004]. This statement also applies to crests and troughs. The height of the crest Cr is the maximum value of the surface elevation $\eta(t)$ between the moment when it crosses the zero level from bottom to top and the moment when it crosses this level from top to bottom [*Forristall*, 2000]. Similarly, the depth of the trough Th is determined. The depth of the trough is the absolute value of the minimum value $\eta(t)$ between two successive intersections of the zero level from top to bottom and from bottom to top.

In the linear model, the probability density functions of random variables Cr and Th coincide and are described by the Rayleigh distribution, which has the form

$$F_R(x) = \frac{x}{x_0^2} \exp\left(-\frac{x^2}{2x_0^2}\right), x \geq 0,$$

where x_0 is scale parameter.

For waves propagating in deep water, a simplified second-order nonlinear model can be used in the narrowband spectrum approximation, which is described by the amplitude-modulated Stokes wave equation

$$\eta(x,t) = a_r(x,t)\cos\theta + \frac{1}{2} k_p a_r^2(x,t)\cos(2\theta), \quad (13)$$

where $a_r(x,t)$ is a wave envelope. The local maxima of the second term in (13) coincide with the crest and trough of the linear wave, so the maximum values of the crest and trough are respectively equal to $Cr_{\max} = a_r + \frac{1}{2}k_p a_r^2$, $Th_{\max} = a_r - \frac{1}{2}k_p a_r^2$, Thus, in the second-order nonlinear model, the peaks are higher and the troughs shallower than predicted by linear theory [*Toffoli et al.*, 2008].

Let us compare the distributions of the crest and trough calculated from the measurement data with the model prediction. To use the measurement data obtained in different situations, we introduce normalization

$$\tilde{\eta}(t) = \frac{\eta(t)}{H_S}.$$

Probability density functions F_{Cr} and F_{Th} depend on the range A_η for which they are calculated [*Zapevalov*, 2024]. As can be seen from Figure 5, the probability density functions F_{Cr} and F_{Th} for $A_\eta > 0$ correspond to a second-order nonlinear model. Contradictions arise when $A_\eta < 0$. In this case, when x is greater than the distribution mode, the inequality $F_{Th} > F_R$ holds. Negative values of skewness are observed much less frequently than positive values [*Zapevalov and Garmashov*, 2022]. Therefore, if we calculate the averages over the full range A_η functions F_{Cr} and F_{Th} then they are consistent with a second-order nonlinear model.

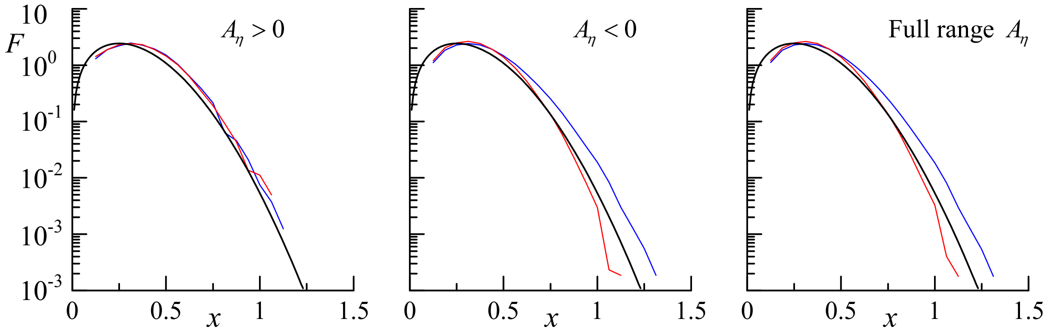

Figure 5. Probability density functions F_{Cr} and F_{Th} calculated for three ranges of skewness A_η. The blue curve is F_{Cr}, the red curve is F_{Th}, the black curve is F_R.

Conclusion

A wide range of fundamental and applied problems are concerned with the description and prediction of nonlinear effects in sea surface waves. Verification of statistical models based on wave measurement data showed the following. Models based on the decomposition of the wave profile by wave steepness, as well as models based on multidimensional integrals of wave spectra, do not describe changes in the skewness and excess kurtosis of sea surface elevations in the range where these parameters change at sea. These models can only describe average dependencies on the wave steepness or inverse waves age in a limited range ($A_\eta > 0$, $E_\eta > 0$). This is not sufficient for their use in engineering applications, where load calculations require information about the limits of changes in parameters characterizing the impact of waves on an object.

The non-linearity of sea waves is determined by several physical mechanisms; therefore their correct description requires the construction of multi-parametric models.

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