Russian Journal of Earth Sciences

Simulation of Infrasound Spectrum Generated by Sea Surface Waves

A. S. Zapevalov*

Marine Hydrophysical Institute, Russian Academy of Sciences, Sevastopol, Russia
* **Correspondence to:** Alexander Zapevalov, sevzepter@mail.ru

Abstract: Sea surface waves create pressure waves of the infrasound range that do not fade with depth, which have a noticeable effect on the processes occurring in the earth's crust. The connection between the energy of surface waves and microseisms makes it possible to solve the inverse problem and reconstruct wave characteristics based on seismic measurement data. These studies require information on the physical factors that cause ambiguity in the relationship between wave spectra and the spectra of infrasound generated by them. In this paper, the change in the shape of the infrasound spectrum is analyzed within the framework of numerical simulation. Well-known surface sea wave spectral models are used for analysis. It is shown that the main factors influencing the shape and peak value of the infrasound spectrum are the difference in the frequencies of the spectral peaks of swell ω_{w1} and wind waves ω_{w2}, as well as the change in the angle between the directions of their propagation. For the same spectrum of surface waves, when $\omega_{w1} \approx \omega_{w2}$, the maximum value of the infrasound spectrum peak occurs at the opposite direction of wave propagation, it decreases by more than 5 times when the directions are mutually orthogonal, it decreases by two orders of magnitude, when the directions coincide. With an increase in the difference between ω_{w1} and ω_{w2}, the frequency range where the generation of infrasound is determined by the interaction of swell and wind waves narrows.

Keywords: sea surface, waves, generation of infrasound, infrasound spectrum.

Citation: Zapevalov A. S. (2025), Simulation of Infrasound Spectrum Generated by Sea Surface Waves, *Russian Journal of Earth Sciences*, 25, ES5009, EDN: DNFNSJ, https://doi.org/10.2205/2025es001006

Research Article

Received: February 3, 2025
Accepted: March 17, 2025
Published: September 29, 2025

Copyright: © 2025. The Author. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

Introduction

Currently, much attention is being paid to research related to the transfer of energy from sea waves to the earth's crust [*Chupin and Gusev*, 2022; *Wilson*, 2018; *Zapevalov*, 2023]. In the linear approximation, the pressure pulsations created by surface waves quickly attenuate with depth and can be neglected at depths greater than half the wavelength. If the depth is small compared to the wavelength, the impact on the seabed can be quite large and generate microseisms of the first kind, with the frequency generating their waves. The generation of infrasound pressure waves (further infrasound) that do not decay with depth is modeled using the framework of three-wave interactions between one acoustic wave and two surface waves propagating in opposite directions. The three-wave interaction leads to the generation of microseisms of the second kind, with twice the frequency of the waves generating them. In this model, the propagation of acoustic waves is described by a linear wave equation, the nonlinearity is associated with boundary conditions for hydrodynamic equations [*Naugolnikh and Rybak*, 2003]. The field of surface waves is represented as a superposition of linear waves [*Brekhovskikh*, 1966a; *Hasselmann*, 1963; *Zapevalov and Pokazeev*, 2016].

Data from wave and seismic measurements indicate that there is a statistical relationship between the characteristics of waves and microseisms. This allows us to solve the inverse problems: to restore the trajectory and speed of movement of typhoons [*Davy et al.*, 2014; *Dolgikh and Mukomel*, 2004], to estimate the energy of surface waves based on seismic measurements [*Bromirski et al.*, 1999; *Donne et al.*, 2014], as well as predicting the arrival of storm waves that have arisen in remote regions. [*Rindraharisaona et al.*, 2020]. At the same time, there were situations when the energy of surface waves calculated using statistical models turned out to be significantly underestimated [*Cutroneo et al.*, 2021]. A variety of spectral characteristics of infrasound microseisms recorded during the passage of cyclones in the Sea of Japan was also noted [*Chupin and Gusev*, 2022].

Situations in which surface waves propagate in opposite directions can be divided into three groups [*Ardhuin et al.*, 2012]. The first situation is when there is one wave system on the sea surface, with a fairly wide distribution of wave energy in directions. This situation was considered in [*Zapevalov*, 2007], where it was shown, that at frequencies close to the spectral peak frequency, the main contribution to infrasound generation is made by waves that propagate in directions orthogonal to the wind direction. The second situation occurs when waves run onto the coast and are reflected from it [*Ardhuin et al.*, 2011]. In this situation, the frequencies of the spectral peaks of the incident and reflected waves coincide. The third is a situation in which two wave systems are simultaneously present on the sea surface. It occurs, for example, when swells coming from the cyclone region interact with waves generated by the local wind [*Tabulevich et al.*, 2001]. The frequencies of the spectral peaks of the swell and wind waves differ.

In this paper, we analyze the spectrum of infrasound that generates microseisms of the second kind. We will focus on the third situation, in which discrepancies in the frequencies of the peaks of the wave spectra, as well as a change in the angle between the directions of wave propagation in two systems, can cause to ambiguity in the statistical relationship between wave and seismic characteristics.

Infrasound Spectrum Model

Two surface waves create a acoustic wave that does not decay with depth and propagates along the normal to the undisturbed surface if the surface waves satisfy the conditions

$$\begin{aligned} \vec{k}_1 &= -\vec{k}_2, \\ \omega_1 &= \omega_2, \end{aligned} \tag{1}$$

where $\vec{k}$ is the wave vector, ω is the angular frequency. We will assume that the field of surface waves is formed by gravitational waves propagating in deep water, which satisfy the dispersion relation

$$\omega^2 = gk, \tag{2}$$

where g is the gravitational acceleration, k is the wave number. The dispersion relation (2) uniquely relates the parameters k and ω, therefore, to generate infrasound, it is necessary and sufficient to satisfy condition (1). If equality (1) is approximate, then the horizontal component of the wave vector of the pressure wave $\vec{K}$ is not zero $\vec{K} = \vec{k}_1 + \vec{k}_2 \neq 0$ and this wave propagates at an angle to the vertical [*Brekhovskikh*, 1966b].

In a linear wave field, which is a superposition of sinusoidal waves, the pressure spectrum is determined by the directional spectrum of surface waves Let us represent the directional spectrum of surface waves as follows

$$F_w(\omega_w, \theta) = S_w(\omega_w)\Theta(\omega_w, \theta),$$

where ω_w is the surface wave frequency; θ is azimuth angle; $S_w(\omega_w)$ is one–dimensional frequency wave spectrum; $\Theta(\omega_w, \theta)$ is spreading function. The spreading function $\Theta(\omega_w, \theta)$ describes the distribution of wave energy across different directions and satisfies the normalization condition

$$\int_0^{2\pi} \Theta(\omega_w, \theta) d\theta = 1. \tag{3}$$

In this formulation of the problem, the infrasound spectrum is related to the wave spectrum by the equation [*Farrell and Munk*, 2013]

$$S_p\left(\omega_p\right) = \frac{\pi}{2}\left(\frac{\rho g}{c}\right)^2 \omega_p^3 S_w^2(\omega_w) I_\Theta(\omega_w),$$

where the infrasound frequency ω_p and the surface wave frequency ω_w are related as $\omega_p = 2\omega_w$; ρ is the density of sea water; C is the speed of sound in water; I_Θ is a factor determined by the angular distribution of wave energy. The multiplier I_Θ has the form

$$I_\Theta(\omega_w) = \int_0^{2\pi} \Theta(\omega_w, \theta)\Theta(\omega_w, \theta + \pi) d\theta. \tag{4}$$

Directional Spectrum of Surface Waves

Let us consider a field of surface waves, which is a superposition of two wave systems: swell (the first system) and wind waves (the second system). In this case, the frequency-angular spectrum can be represented as

$$F_{w\Sigma}(\omega_w, \theta) = S_{w\Sigma}(\omega_w)\Theta_\Sigma(\omega_w, \theta) = S_{w1}(\omega_w)\Theta_1(\omega_w, \theta) + S_{w2}(\omega_w)\Theta_2(\omega_w, \theta),$$

Here and below, the subscripts Σ, 1 and 2 respectively indicate that this characteristic refers to the resulting wave field, swell and wind waves. The frequency spectrum and the spreading function are given as

$$\begin{aligned} S_{w\Sigma}(\omega_w) &= S_{w1}(\omega_w) + S_{w2}(\omega_w). \\ \Theta_\Sigma(\omega_w, \theta) &= R_1(\omega_w)\Theta_1(\omega_w, \theta) + R_2(\omega_w)\Theta_2(\omega_w, \theta), \end{aligned} \tag{5}$$

where $R_1(\omega_w) = S_{w1}(\omega_w) / S_{w\Sigma}(\omega_w)$; $R_2(\omega_w) = S_{w2}(\omega_w) / S_{w\Sigma}(\omega_w)$. Spreading functions $\Theta_\Sigma(\omega_w, \theta)$, $\Theta_1(\omega_w, \theta)$ and $\Theta_2(\omega_w, \theta)$ satisfy the normalization condition (3). For certainty, we assume that the frequency of the spectral peak of the first wave system is lower than the frequency of the spectral peak of the second wave system.

In the frequency domain, where the functions $R_1(\omega_w)$ or $R_2(\omega_w)$ of one of the wave systems is close to one, the influence of the other wave system can be ignored. In this frequency domain, the generation of infrasound will be determined only by the spreading function of a single wave system.

We will use an approximation of the frequency spectrum of the swell in the form of the JONSWAP spectrum [*Lucas and Soares*, 2015]

$$S_{w1}(\omega_w) = A_1\left(\frac{\omega_w}{\omega_{w1}}\right)^{-5} \exp\left(-\frac{5}{4}\left(\frac{\omega_w}{\omega_{w1}}\right)^{-4}\right)\Gamma^\Phi,$$

where A is constant depending on the height of the waves; ω_{w1} is the frequency of the peak of the swell spectrum; $\Gamma = 1.7$; $\Phi = \exp\left(-0.5(\omega_w - \omega_{w1})^2 \upsilon^{-2} \omega_{w1}^{-2}\right)$; $\upsilon = 0.07$ when $\omega \le \omega_{w1}$, $\upsilon = 0.09$ when $\omega > \omega_{w1}$.

The spectrum of wind waves is given in the form of a modified JONSWAP spectrum proposed in the paper [*Donelan et al.*, 1985]. This spectrum is also called the Donelan spectrum. The parameters of this model are the frequency of the peak of the wind wave spectrum ω_{w2} and the parameter ς, which characterizes the stage of development of the wave field. The parameter , called the inverse wave age, is defined as

$$\varsigma = \frac{W_{10}}{c_{w2}},$$

where c_{w2} is phase velocity of wind waves with frequency ω_{w2}. Large values of the parameter correspond to an earlier stage of development of the wave field. The boundary value of the parameter ς separating wind waves and swell is 0.83. The Donelan spectrum has the form

$$S_{w2}(\omega_w) = A_2 \omega_w^{-4} \exp\left\{ -\left(\frac{\omega_{w2}}{\omega_w}\right)^4 \right\} \Gamma^{\Phi},$$

where $\Phi = \exp\left(-0.5(\omega_w - \omega_{w2})^2 \upsilon^{-2} \omega_{w2}^{-2}\right)$; $\upsilon = 0.08\left(1 + \frac{4}{\varsigma^3}\right)$; $\Gamma = 1.7 + 6.0\, lg(\varsigma)$.

Let the frequencies of the spectral peaks of two wave systems be related as $\omega_{w2} = \gamma \omega_{w1}$. It is advisable to move to dimensionless frequency by introducing normalization

$$\widetilde{\omega}_w = \frac{\omega_w}{\omega_{w1}}, \tag{6}$$

from which it follows $\widetilde{\omega}_{w1} = 1$, $\widetilde{\omega}_{w2} = \gamma = \frac{\omega_{w2}}{\omega_{w1}}$. Further we will assume that $S_{w1}(\omega_{w1}) = S_{w2}(\omega_{w2})$. To compare spectra calculated in different situations, we introduce a normalization in which the maximum value of the spectrum of each wave system is equal to one. The spectra of each wave system normalized in this way and the total spectrum calculated for them will be denoted by the tilde symbol.

As can be seen from Figure 1, the spectrum $\widetilde{S}_{\Sigma}(\omega_w)$ and functions $R_1(\omega_w)$, $R_2(\omega_w)$ strongly depend on the ratio of the frequencies of the peaks of the spectra of wind waves and swell. Changes in the functions $R_1(\omega_w)$ and $R_2(\omega_w)$ indicate that with an increase in the parameter γ, the frequency domain where the generation of infrasound is determined by the interaction of wind waves and swells narrows, and the areas where the generation of infrasound is determined by the angular distribution of only one wave system expands. Also, as the parameter γ increases, the spectrum $\widetilde{S}_{w\Sigma}$ becomes more broadband, and its maximum decreases.

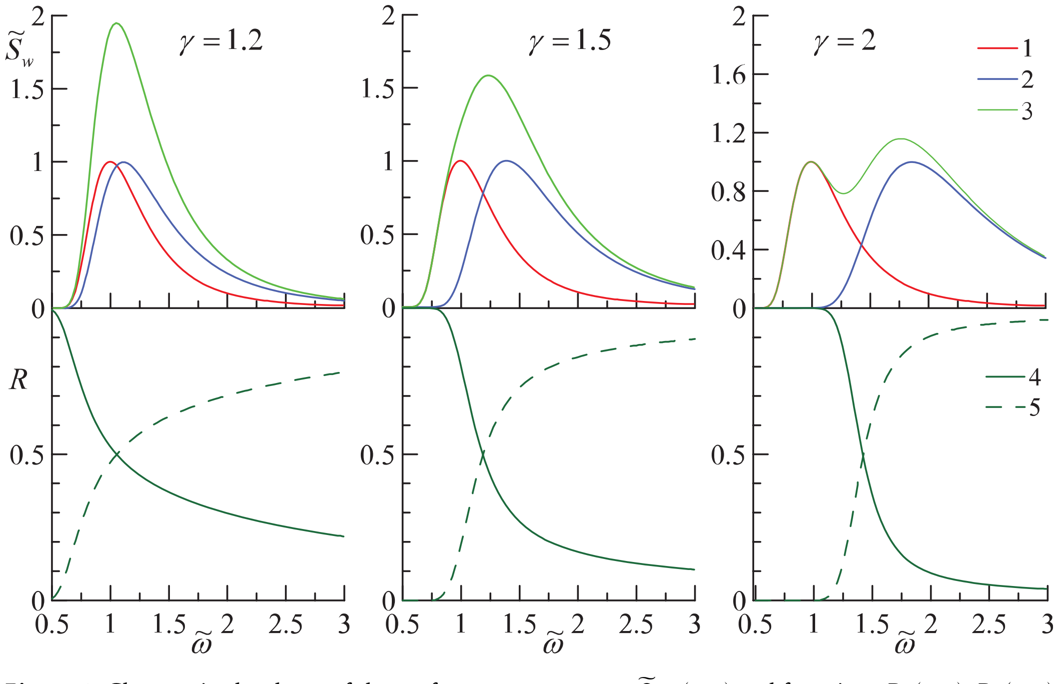

Figure 1. Changes in the shape of the surface wave spectrum $\widetilde{S}_{w\Sigma}(\omega_w)$ and functions $R_1(\omega_w)$, $R_2(\omega_w)$ with a change in the parameter γ. Curves 1–3 are $\widetilde{S}_{w1}$, $\widetilde{S}_{w2}$, $\widetilde{S}_{w\Sigma}$; curves 4 and 5 are R_1 and R_2 respectively.

The spreading functions $\Theta(\omega_w, \theta)$ are most narrowly directed at frequencies near the spectral peak and expand towards low and high frequencies [*Babanin and Soloviev*, 1998; *Donelan et al.*, 1985; *Mitsuyasu et al.*, 1975]. For wind waves and swell, we define a spreading function in the form [*Hasselmann et al.*, 1980]

$$\Theta(\omega_w, \theta) = N \cos^{2s}\left(\frac{\theta - \theta_0}{2}\right),$$

where N is the normalizing factor determined from the condition (3); θ_0 is the angle corresponding to the main direction of wave propagation;

$$s = \begin{cases} 9.77\left(\dfrac{\omega_w}{\omega_{w0}}\right)^{-2.33-1.45(\varsigma-1.17)}, & \dfrac{\omega_w}{\omega_{w0}} > 1.05 \\ 6.97\left(\dfrac{\omega_w}{\omega_{w0}}\right)^{4.06}, & \dfrac{\omega_w}{\omega_{w0}} < 1.05 \end{cases}.$$

Here for the swell $\omega_{w0} = \omega_{w1}$, for wind waves $\omega_{w0} = \omega_{w2}$.

Modeling of the Infrasound Spectrum

In a wave field where two wave systems are present, the distribution of wave energy in directions is described by spreading function (5). The multiplier I_Θ determining the energy of waves propagating in the opposite direction becomes multiparametric $I_\Theta = I_\Theta(\omega_w, \omega_{w1}, \omega_{w2}, \theta_1, \theta_2)$. Since the level of infrasound generated by surface waves depends on the angle between the main directions of wave propagation, it is advisable to introduce a parameter $\Delta\theta = \theta_1 - \theta_2 - \pi$. Taking into account the normalization (6), we obtain $I_\Theta = I_\Theta(\widetilde{\omega}_w, \gamma, \Delta\theta)$.

To analyze the changes in the shape of the infrasound spectra calculated for different values of the parameter $\Delta\theta$, all spectra were normalized to $\max S_p\left(\omega_p, \Delta\theta = 0\right)$. The change in the shape of the infrasound spectrum and the multiplier I_Θ at a fixed value of parameter γ and different mutual directions of the two wave systems are shown in Figure 2.

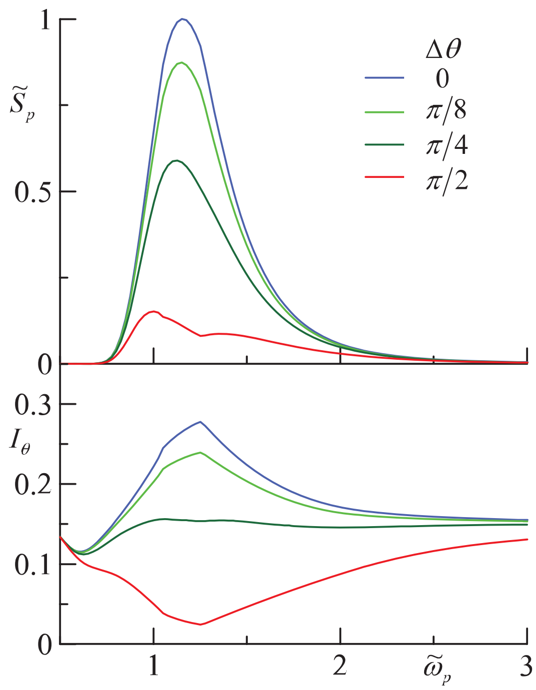

Figure 2. Changing the shape of the infrasound spectrum $\widetilde{S}_p$ and multiplier I_θ with different mutual directions of the two wave systems.

Calculations were performed for the case when the peak frequencies of two wave systems are sufficiently close $\gamma = 1.2$. With increasing $\Delta\theta$, the maximum value of the infrasound spectrum rapidly decreases, reaching a minimum at $\Delta\theta = \pi$, i.e. in a situation where the main directions of propagation of swell and wind waves coincide.

Let us analyze how the maximum value of the infrasound spectrum changes with other ratios of frequencies ω_{w1} and ω_{w2}. To do this, we consider a family of functions

$$\Psi(\Delta\theta, \gamma) = \frac{\max\left(S_p(\Delta\theta, \gamma = \text{const})\right)}{\max\left(S_p(\Delta\theta = 0, \gamma = \text{const})\right)}.$$

As follows from Figure 3, with an increase in the parameter γ, the general appearance of the function $\Psi(\Delta\theta, \gamma)$ is preserved, maximum at $\Delta\theta = 0$, minimum at $\Delta\theta = \pi$. For large values γ, the function $\Psi(\Delta\theta, \gamma)$ weakly depends on $\Delta\theta$, and its minimum value approaches unity. At $\gamma = 5$ the minimum value of the function $\Psi(\Delta\theta, \gamma)$ is 0.9. The degeneration of the function $\Psi(\Delta\theta, \gamma)$ into a constant close to unity means that in this case the infrasound spectrum at frequencies close to the maximum frequency of the spectrum of one of the two surface wave systems can be calculated without taking into account the presence of the other.

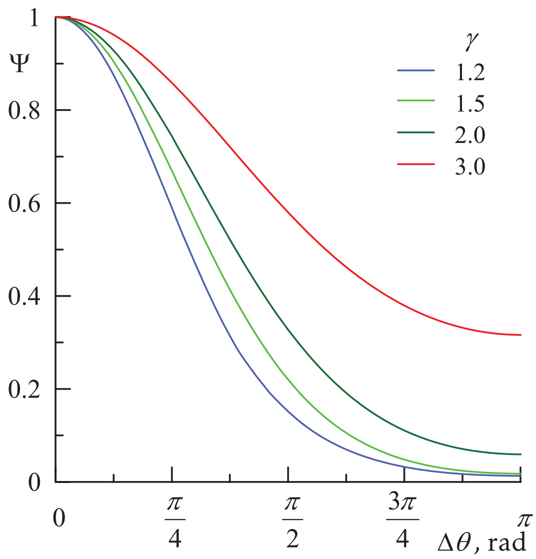

Figure 3. Family of functions $\Psi(\Delta\theta, \gamma)$.

Conclusion

Numerical analysis performed within the framework of well-known models of frequency spectra and spreading functions of the angular distribution of sea waves makes it possible to better understand the mechanism of formation of the infrasound spectrum by the sea surface. In a situation where two wave systems (swell and wind waves) are present on the sea surface, the infrasound spectrum significantly depends on the angle between the main directions of their propagation and on the frequency difference of their spectral peaks. If the condition $\omega_{w1} \approx \omega_{w2}$ is met, then for the same spectrum of surface waves, the maximum value of the peak of the infrasound spectrum occurs with the opposite main direction of wave propagation, with a mutually orthogonal main direction it decreases by more than 5 times, and with the coincidence of main directions it decreases by two orders of magnitude. If the difference ω_{w1} and ω_{w2} increases, then the frequency domain within which the generation of infrasound is determined by the interaction of swell and wind waves narrows. When the condition $\omega_{w2} / \omega_{w1} > 5$ is met, the generation of infrasound around ω_{w1} is determined mainly by the spreading function of swell, around ω_{w2} is determined by the spreading function of wind waves.

Acknowledgements. The work was completed within the framework of the state assignment on the topic FNNN-2024-0012 "Analysis, diagnosis and real-time forecast of the state of hydrophysical and hydrochemical fields of marine water areas based on mathematical modeling using data from remote and in situ methods of measurements".

References

Ardhuin F., Balanche A., Stutzmann E., et al. From seismic noise to ocean wave parameters: General methods and validation // Journal of Geophysical Research: Oceans. — 2012. — Vol. 117, no. C5. — https://doi.org/10.1029/2011jc007449.

Ardhuin F., Stutzmann E., Schimmel M., et al. Ocean wave sources of seismic noise // Journal of Geophysical Research. — 2011. — Vol. 116, no. C9. — https://doi.org/10.1029/2011jc006952.

Babanin A. V. and Soloviev Yu. P. Variability of directional spectra of wind-generated waves, studied by means of wave staff arrays // Marine and Freshwater Research. — 1998. — Vol. 49, no. 2. — P. 89–101. — https://doi.org/10.1071/mf96126.

Brekhovskikh L. M. On the generation of sound waves in a liquid by surface wave // Akusticheskij Zhurnal. — 1966a. — Vol. XII, no. 3. — P. 376–379.

Brekhovskikh L. M. Sound waves under water caused by surface waves in the ocean // Izvestiya AN SSSR. Fizika atmosfery i okeana. — 1966b. — Vol. 2, no. 9. — P. 970–980. — (In Russian).

Bromirski P. D., Flick R. E. and Graham N. E. Ocean wave height determined from inland seismometer data: Implications for investigating wave climate changes in the NE Pacific // Journal of Geophysical Research: Oceans. — 1999. — Vol. 104, no. C9. — P. 20753–20766. — https://doi.org/10.1029/1999jc900156.

Chupin V. A. and Gusev E. S. Infrasound oscillations caused by extratropical cyclones in the sea of Japane // Hydrosphere. Hazard processes and phenomena. — 2022. — Vol. 3, no. 4. — P. 346–354. — https://doi.org/10.34753/HS.2021.3.4.346. — (In Russian).

Cutroneo L., Ferretti G., Barani S., et al. Near Real-Time Monitoring of Significant Sea Wave Height through Microseism Recordings: Analysis of an Exceptional Sea Storm Event // Journal of Marine Science and Engineering. — 2021. — Vol. 9, no. 3. — P. 319. — https://doi.org/10.3390/jmse9030319.

Davy C., Barruol G., Fontaine F. R., et al. Tracking major storms from microseismic and hydroacoustic observations on the seafloor // Geophysical Research Letters. — 2014. — Vol. 41, no. 24. — P. 8825–8831. — https://doi.org/10.1002/2014gl062319.

Dolgikh G. I. and Mukomel D. V. Dependence of microseism variation periods upon the cyclone propagation velocity and direction // Doklady Earth Sciences. — 2004. — Vol. 394, no. 1. — P. 141–144.

Donelan M. A., Hamilton J. and Hui W. H. Directional spectra of wind-generated waves // Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences. — 1985. — Vol. 315, no. 1534. — P. 509–562. — https://doi.org/10.1098/rsta.1985.0054.

Donne S., Nicolau M., Bean C., et al. Wave height quantification using land based seismic data with grammatical evolution // 2014 IEEE Congress on Evolutionary Computation (CEC). — IEEE, 2014. — P. 2909–2916. — https://doi.org/10.1109/cec.2014.6900563.

Farrell W. E. and Munk W. Surface gravity waves and their acoustic signatures, 1-30 Hz, on the mid-Pacific sea floor // The Journal of the Acoustical Society of America. — 2013. — Vol. 134, no. 4. — P. 3134–3143. — https://doi.org/10.1121/1.4818780.

Hasselmann D. E., Dunckel M. and Ewing J. A. Directional Wave Spectra Observed during JONSWAP 1973 // Journal of Physical Oceanography. — 1980. — Vol. 10, no. 8. — P. 1264–1280. — https://doi.org/10.1175/1520-0485(1980)010<1264:dwsodj>2.0.co;2.

Hasselmann K. A statistical analysis of the generation of microseisms // Reviews of Geophysics. — 1963. — Vol. 1, no. 2. — P. 177–210. — https://doi.org/10.1029/RG001i002p00177.

Lucas C. and Soares G. Guedes. On the modelling of swell spectra // Ocean Engineering. — 2015. — Vol. 108. — P. 749–759. — https://doi.org/10.1016/j.oceaneng.2015.08.017.

Mitsuyasu H., Tasai F., Suhara T., et al. Observations of the Directional Spectrum of Ocean Waves Using a Cloverleaf Buoy // Journal of Physical Oceanography. — 1975. — Vol. 5, no. 4. — P. 750–760. — https://doi.org/10.1175/1520-0485(1975)005<0750:ootdso>2.0.co;2.

Naugolnikh K. A. and Rybak S. A. Sound generation due to the interaction of surface waves // Acoustical Physics. — 2003. — Vol. 49, no. 1. — P. 88–90. — https://doi.org/10.1134/1.1537393.

Rindraharisaona E. J., Cordier E., Barruol G., et al. Assessing swells in La Réunion Island from terrestrial seismic observations, oceanographic records and offshore wave models // Geophysical Journal International. — 2020. — Vol. 221, no. 3. — P. 1883–1895. — https://doi.org/10.1093/gji/ggaa117.

Tabulevich V. N., Ponomarev E. A., Sorokin A. G., et al. Standing sea waves, microseisms, and infrasound // Izvestiya, Atmospheric and Oceanic Physics. — 2001. — Vol. 37, no. 2. — P. 218–226.

Wilson J. D. Modeling Microseism Generation by Inhomogeneous Ocean Surface Waves in Hurricane Bonnie Using the Non-Linear Wave Equation // Remote Sensing. — 2018. — Vol. 10, no. 10. — P. 1624. — https://doi.org/10.3390/rs10101624.

Zapevalov A. S. The effect of anisotropy of a rough sea surface on the generation of acoustic radiation // Acoustical Physics. — 2007. — Vol. 53, no. 1. — P. 75–79. — https://doi.org/10.1134/s1063771007010095.

Zapevalov A. S. Impact of the sea waves' skewness and group structure on the infrasound generation by the sea surface // Physical Oceanography. — 2023. — Vol. 30, no. 2. — P. 160–170. — https://doi.org/10.29039/1573-160X-2023-2-160-170.

Zapevalov A. S. and Pokazeev K. V. Modeling the spectrum of infrasonic hydroacoustic radiation generated by the sea surface under storm conditions // Acoustical Physics. — 2016. — Vol. 62, no. 5. — P. 554–558. — https://doi.org/10.1134/S1063771016050195.